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FPF RINGS CHARACTERIZED BY TWO-GENERATED FAITHFUL MODULES

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0. Introduction

Azumaya [2], Osofsky [20] and Utumi [26] considered various properties of QF rings, e.g., the completely faithfulness (i.e., generator) of modules, the injective cogenerator and Morita duality. They introduced and studied one new class of, so-called, *right PF (pseudo-Frobenius) rings*, i.e., rings R whose every faithful right R-module is a generator for Mod-R, the category of all right R-modules. Afterward, Endo [6] and Tachikawa [25] naturally studied (commutative noetherian or perfect) rings R satisfying the condition that every *finitely generated* faithful right R-module is a generator for Mod-R. Rings satisfying this condition are called *right FPF (finitely pseudo-Frobenius) rings* whose general studies were made, at first over semiperfect rings, by Faith [7], [8]. The study of commutative or semiperfect FPF rings was improved in more detail ([9], [10] [11], [13], [14], [21]). Most of the basic results on FPF rings may be found in Faith and Page [12].

We now consider, for each positive integer n, the condition "right n-PF" on a ring R that every n-generated (i.e., generated by at most n elements) faithful right R-module is a generator for Mod-R ([27]). Thus the rings that are right n-PF for all positive integers n are just the right FPF rings, and there exists a chain of conditions:

$$\text{FPF} \Rightarrow \cdots \Rightarrow (n+1)\text{-PF} \Rightarrow n\text{-PF} \Rightarrow \cdots \Rightarrow 1\text{-PF}.$$

Concerning this, it was shown in [4], [17] that a right self-injective ring is right FPF if and only if it is right 1-PF, while a commutative semiprime or von Neumann regular ring is right FPF if and only if it is right 2-PF. We then ask generally whether the chain of conditions from FPF to n-PF for some positive integer n collapses to a single condition, i.e., n-PF $\Rightarrow$ FPF. Obviously, 1-PF does not imply FPF in general (for every commutative ring is 1-PF). Thus it is natural to ask whether 2-PF implies FPF. We do not know whether this is true in general case.

In this paper, we shall study semiperfect or commutative FPF rings in connection with the noted above. In Section 1, we present a characterization of semiperfect FPF rings, which shows that 2-PF $\Rightarrow$ FPF for semiperfect rings. Section 2 is concerned with commutative FPF rings. In the section, we characterize these rings R by over-