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Osaka J. Math.
35 (1998), 835–854

BLOCKS OF FACTOR GROUPS AND HEIGHTS OF CHARACTERS

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(Received May 9, 1997)

Introduction

Let G be a finite group and p a prime number. Let (K, R, k) be a p-modular system. We assume that K contains a primitive $|G|$-th root of unity and that k is algebraically closed. Let ν be the valuation of K normalized so that $\nu(p) = 1$. Let N be a normal subgroup of G and let V be an indecomposable oG-module such that V_N is indecomposable, where $o = R$ or k. As in [14], we say that a block B of G V-dominates a block $\overline{B}$ of G/N if there is an $o[G/N]$-module X in $\overline{B}$ such that $V \otimes \mathrm{Inf}X$ belongs to B, where $\mathrm{Inf}X$ denotes the inflation of X to G. In [14] we have shown that there is a natural relation between B and $\overline{B}$, if B V- dominates $\overline{B}$. In particular, if D is a defect group of B, then $\overline{B}$ has a defect group of the form QN/N with $D \cap N \leq Q \leq D$. Then, we shall show in Section 2 that Q chosen in this way is of a rather restricted nature. In fact, we see that $O_p(N_G(Q)) = Q$ and that Q is a Sylow intersection in G (Theorem 2.1). When, for example, V_N is irreducible, there exists a B-Brauer pair (Q, b_Q) (Theorem 2.8). As a consequence, we see there exist defect groups D and $\overline{D}$ respectively of B and $\overline{B}$ such that $Z(D)N/N \leq \overline{D} \leq DN/N$. Further, Q is then a "defect intersection". When V is the trivial module "V-domination" is nothing but the usual "domination", in which case we shall show even the existence of a weight (Q, S) belonging to B (in the sense of Alperin [2]) (Proposition 2.6).

In Section 1 we give an alternative proof of a result of Harris-Knörr [8].

In Section 3 we give an extendibility theorem for an irreducible character of a normal subgroup, the proof of which depends upon a result of Brauer on major subsections [4, (4C)] and a result of Knörr [11, Corollary 3.7 (i)].

As an application we study in Section 4 the following conjecture $(*)$ given by Robinson [17]. In [17] $(*)$ is proved under a conjecture related to Alperin's weight conjecture, cf. Theorem 5.1 in [17].

$(*)$ Let B be a block of a group G with defect group D. Then, for every irreducible character χ in B, $\mathrm{ht}\chi \leq \nu|D : Z(D)|$ and the equality holds only when D is abelian.

The conjecture $(*)$ is of course an extension of half of Brauer's height 0 conjec-