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ERROR ESTIMATE IN OPERATOR NORM OF EXPONENTIAL PRODUCT FORMULAS FOR PROPAGATORS OF PARABOLIC EVOLUTION EQUATIONS

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1. Introduction

In the present work we study an error estimate in the operator norm of exponential product approximation for propagators of parabolic evolution equations. The obtained result applies to Schrödinger operators $-\Delta + V(t,x)$ with a certain class of time dependent singular potentials. One of typical examples is a positive Coulomb potential like $V(t,x) = c/|x-g(t)|$, $c > 0$, which has a singularity moving with time t.

Let X be a Hilbert space and let $\| \ \|$ denote the operator norm of bounded operators acting on X. We are now given positve self-adjoint operators A, $B(t) \geq c > 0$, t being in a compact interval $[0,T]$. We note that the assumption of positivity is not essential. In the discussion below, we have only to assume that these operators are semibounded uniformly in t. If the domain of $B(t)$ fulfills the inclusion relation

$$\mathcal{D}(A^\alpha) \subset \mathcal{D}(B(t)) \tag{1.1}$$

for some α, $0 \leq \alpha < 1$, independent of t, then the sum $C(t) = A + B(t)$ also becomes a positive self-adjoint operator with domain $\mathcal{D}(C(t)) = \mathcal{D}(A)$ independent of t. If, in addition, $B(t)$ satisfies a suitable continuity condition (see assumption (A) below), then $C(t)$ generates the propagator $U(t,s)$, $0 \leq s \leq t \leq T$, to the evolution equation

$$\partial_t U(t,s) = -C(t)U(t,s), \quad U(s,s) = \mathrm{Id},$$

where Id is the identity operator. As is easily seen, $U(t,s) : X \to X$ is a contraction operator.

We now consider the exponential product approximation for the propagator $U(t,0)$. Let

$$0 = t_0 < t_1 < \ldots < t_{N-1} < t_N = t, \quad t_j = j\tau, \quad \tau = t/N,$$