EXISTENCE OF INVARIANT MEASURES
FOR DIFFUSION PROCESSES ON A WIENER SPACE

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1. Introduction

In this paper, we discuss existence of finite invariant measures for diffusion processes with unbounded drift on an abstract Wiener space \((B, H, \mu)\). The processes we treat are the ones which have generators formally expressed as \(-\frac{1}{2}D^* \sigma D + (b, D)\), where \(D\) stands for the \(H\)-derivative in the sense of Malliavin calculus, \(D^*\) the adjoint operator of \(D\) with respect to the Wiener measure \(\mu\), \(\sigma\) a diffusion coefficient, being a function on \(B\) taking values in the space of positive symmetric operators on \(H\), and \(b\) a drift coefficient, being an \(H\)-valued function on \(B\). When \(\sigma \equiv \) identity, a given generator has rigorous meaning in \(L^2\) sense under a mild condition on \(b\), and existence of the associated diffusion and invariant measures was proved by Shigekawa [17] in the case that \(b\) is bounded. Partially generalized cases were treated by Vintschger [21] and Zhang [22]. We extend these results to the case where \(\sigma\) is not constant and \(b\) is not necessarily bounded but has only some kind of exponential integrability. The diffusion process is constructed by using the theory of Dirichlet forms and the Girsanov transformation, so measures charging no set of zero capacity are allowed to be initial measures. In this paper, however, we restrict our attention to absolutely continuous measures with respect to \(\mu\), as in the papers mentioned above. That is, our formulation is as follows: Let \(\{T_t\}\) be the associated Markovian semigroup on \(L^\infty(\mu)\). Is there any non-zero \(\rho \in L^1(\mu)\) such that

\[
\int_B (T_t f) \rho \, d\mu = \int_B f \rho \, d\mu \quad \text{for every } t > 0 \text{ and } f \in L^\infty(\mu)?
\]

If there is, then \(\rho \, d\mu\) is an invariant measure of \(\{T_t\}\), and of the diffusion process.

Note that, in a different formulation from ours, invariant measures are proved to be absolutely continuous with respect to \(\mu\); see [3, 2].

The difficulty in proving existence of invariant measures lies in the point that the resolvent of the generator is no more compact and that spectrums are not necessarily eigenvalues. So, as in [17], first we consider the case when \(B\) is finite dimensional, then take approximation in general cases. At this point, the property of hyperbound-

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