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INCOMPRESSIBILITY OF CLOSED SURFACES IN TOROIDALLY ALTERNATING LINK COMPLEMENTS

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1. Introduction

It has been known that alternating knots and links have many nice properties, and there have been many generalizations of the notion of alternating links. For instance, homogeneous, adequate, augmented alternating and almost alternating links. C.C. Adams introduced the notion of toroidally alternating links in [1]. The class of toroidally alternating links turned out to contain those of alternating links, almost alternating links, pretzel links and Montesinos links (See [1] and [3]). C. Hayashi also studied alternating links on surfaces of arbitrary positive genera in [3].

The purpose of this paper is to determine incompressibility of given closed orientable surfaces in toroidally alternating link complements.

Let M be a lens space or the 3-sphere S^3, and let T be a torus which gives a Heegaard splitting of M. This torus is unique up to isotopy (See [2]). A link L in M is called *toroidally alternating* (*with respect to* T) if it can be isotoped into a neighborhood $T \times I$ of T so that it has an alternating diagram $\pi(L)$ on T such that $T - \pi(L)$ consists of open discs with respect to a projection $\pi : T \times I \to T$.

Let $F \subset M - L$ be an embedded connected closed surface. F is called *incompressible* in $M - L$ if either F is a 2-sphere and F does not bound a 3-ball in $M - L$, or F is not a 2-sphere and for each disc $D \subset M - L$ with $D \cap F = \partial D$, there is a disc $D' \subset F$ with $\partial D = \partial D'$. F is called *pairwise incompressible* if there does not exist a disc $D \subset M$ meeting L transversely in one point with $D \cap F = \partial D$. An embedded disconnected surface F in $M - L$ is called *incompressible* in $M - L$ (resp. *pairwise incompressible*) if every component of F is incompressible in $M - L$ (resp. pairwise incompressible).

A link $L \subset M$ is called *non-split* if every 2-sphere embedded in $M - L$ bounds a 3-ball in $M - L$.

W. Menasco defined the notion of *standard position* for surfaces either without boundary or with meridional boundary in alternating knot and link complements

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