Takegoshi, K.
Osaka J. Math.
34 (1997), 783–802

ON COHOMOLOGY GROUPS OF NEF LINE BUNDLES TENSORIZED WITH MULTIPLIER IDEAL SHEAVES ON COMPACT KÄHLER MANIFOLDS

KENSHO TAKEGOSHI

(Received October 25, 1996)

Introduction

Let X be a compact Kähler manifold of dimension n provided with a Kähler metric ω_X and let E be a holomorphic line bundle on X. E is said to be *numerically effective*, "*nef*" for short, if the real first Chern class $c_{R,1}(E)$ of E is contained in the closure of the Kähler cone of X. If X is projective algebraic, then E is nef if and only if $C \cdot E = \int_C c_{R,1}(E) \geq 0$ for any irreducible reduced curve C of X (cf.[13], §2 and [1], §6).

If E is nef, then for a fixed smooth metric h_E of E and a given sequence of positive numbers $\{\varepsilon_k\}_{k\geq 1}$ decreasing to zero, there exists a sequence of real-valued smooth functions $\{\varphi_k\}_{k\geq 1}$ such that every form $\Theta_E + dd^c\varphi_k + \varepsilon_k\omega_X$ yields a Kähler metric. Here Θ_E is the curvature form of E relative to h_E defined by $\Theta_E = dd^c(-\log\, h_E)$ with $d^c = \sqrt{-1}(\bar\partial - \partial)/2$. Normalizing φ_k in such a way that $\sup_X \varphi_k = 0$, we can show that φ_k converges to an integrable function φ_∞ on X so that $\Theta_E + dd^c\varphi_\infty$ is a positive current (cf. §2, Proposition 2.5). Such an integrable function φ_∞ is said to be *almost plurisubharmonic*. In general φ_∞ has singularities and $e^{-\varphi_\infty}$ is not integrable on X (cf. [11], [18]), which implies that the nefness is strictly weaker than the semi-positivity of line bundle in the sense of Kodaira (cf. [4], Example 1.7). Hence we can define a coherent analytic sheaf of ideal $\mathcal{I}(\varphi_\infty)$ associated to φ_∞ whose zero variety (possibly empty) is the set of points in a neighborhood of which $e^{-\varphi_\infty}$ is not integrable. The sheaf $\mathcal{I}(\varphi_\infty)$ is called the *multiplier ideal sheaf* associated to φ_∞ and we obtain the canonical homomorphism $\iota^q(\varphi_\infty) : H^q(X, \mathcal{I}(\varphi_\infty) \bigotimes \Omega^n_X(E)) \longrightarrow H^q(X, \Omega^n_X(E))$ induced by $\iota(\varphi_\infty) : \mathcal{I}(\varphi_\infty) \bigotimes \Omega^n_X(E) \hookrightarrow \Omega^n_X(E)$.

Though φ_∞ can not be uniquely determined generally, the study of $H^q(X, \mathcal{I}(\varphi_\infty) \bigotimes \Omega^n_X(E))$ is deeply related to several interesting problems in analytic and algebraic geometry (cf. [2], [3], [11], [12], [18]). Nevertheless not much is known about the cohomology group except a vanishing theorem for multiplier ideal sheaves associated to nef and big line bundles by Nadel (cf. [11]). We study the cohomology group by establishing a certain harmonic representation theorem. In particular we