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ON RATIONALITY OF LOGARITHMIC $\mathbb{Q}$-HOMOLOGY PLANES-II

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Introduction

This is the second paper in a series of three articles. In the first article, authored jointly by the second and the third author, we showed that a logarithmic $\mathbb{Q}$-homology plane which is non-rational is smooth (See [4]). In this part we will show that there are no $\mathbb{Q}$-homology planes whose projective completion is a surface of general type. As has been observed, due to [3], we need consider only those $\mathbb{Q}$-homology planes with logarithmic Kodaira dimension equal to 2. We shall continue to use notations and results of [4] and refer to it as part I. In fact even the section numbers are continuation of those in part I.

6. Listing of trees

We begin with a smooth, non-rational $\mathbb{Q}$-homology plane V, with $\overline{\kappa}(V) = 2$ and a smooth projective completion X with $\kappa(X) = 2$. Recall that $\Delta = X \setminus V$ is an MNC-divisor, and T is its dual graph. In this section our aim is to give a complete list for T. Subsequently, we eliminate all these possibilities, thus proving that there is no non-rational $\mathbb{Q}$-homology plane as above.

Proposition 1. *The dual graph T of Δ necessarily falls into one of the cases listed in Table* 1. *Entries in the column 'Weight Set' give the weights of vertices of T other than* (-2). T_0 *stands for the tree mentioned in Lemma* 4.4.

Lemma 6.1. *We have* $\theta \leq 2$.

Proof. By (15) we see that $\theta \leq 3$. If possible, let $\theta = 3$. Then again by (15) we see that $\beta_2'' = 9$ and by (12) we have $\nu \geq -1$. Also, by Lemma 4.5 it follows that $\lambda < \theta$.

First consider the case $\lambda = 2$. Then $r_i = 0$ for $i \geq 4$ and

$$\sigma + \tau + e_1 + r_3 + u = 1.$$

[1]Results obtained in sections 6 and 7 are part of C.R. Pradeep's doctoral thesis