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ON AUTOMORPHISMS OF SUPERSINGULAR K3 SURFACES

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1. Introduction

Let k be an algebraically closed field of positive characteristic which we take as the ground field. Let $f : X \to C$ be a morphism from a nonsingular projective surface X to a nonsingular projective curve C such that almost all fibers of f are nonsingular elliptic curves. Furthermore, we assume that f has a section O, which is a morphism from C to X such that $f \circ O = \mathrm{id}_C$. We call such a surface X an elliptic surface over k. The generic fiber E of f is an elliptic curve defined over the function field $K = k(C)$ of C with the K-rational point which we denote by O by abuse of notation. So, we can consider the Mordell-Weil group $E(K)$ consisting of all K-rational points of E which is a finitely generated abelian group with zero element O by a theorem of Lang and Néron. We define the Mordell-Weil group of an elliptic surface $f : X \to C$ as the Mordell-Weil group $E(K)$ of its generic fiber E.

In [5], Miranda and Persson classified all the rational elliptic surfaces over the complex field with finite Mordell-Weil group. W. Lang [3],[4] classified all rational elliptic surfaces over an algebraically closed field of positive characteristic under the hypothesis that the degenerate fibers are all semi-stable. They call these surfaces *extremal* rational elliptic surfaces. In the present paper, we consider extremal elliptic K3 surfaces over an algebraically closed field of characteristic $p \geq 5$. An elliptic surface defined over an algebraically closed field of positive characteristic is called *extremal* if its Mordell-Weil group is finite and its Picard number $\rho(X)$ is equal to the second Betti number $b_2(X)$. An extremal elliptic K3 surface is therefore a supersingular K3 surface in the sense that its Picard number is equal to the second Betti number.

Our main theorem is stated as follows:

Theorem 1.1. *Let k be an algebraically closed field of characteristic $p \geq 5$ and let $f : X \to \boldsymbol{P}^1$ be a supersingular elliptic $K3$ surface defined over k such that the group of sections of f is finite.*

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