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STRUCTURE OF A CLASS OF POLYNOMIAL MAPS WITH INVARIANT FACTORS

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Let $\mathbf{R}[x_1, \cdots, x_n]$ be a ring of polynomials of n $(n \geq 3)$ indeterminates with coefficients in $\mathbf{R}$.

Let $\phi = (\phi_1, \ldots, \phi_n) \in (\mathbf{R}[x_1, \cdots, x_n])^n$ be a polynomial map from $\mathbf{R}^n$ to $\mathbf{R}^n$. Let $x = (x_1, x_2, \ldots, x_n)$ and define

$$\lambda_{n,a}(x) := \sum_{i=1}^{n} x_i^2 - a \prod_{i=1}^{n} x_i \in \mathbf{R}[x_1, x_2, \ldots, x_n].$$

where a $(\neq 0) \in \mathbf{R}$. We will write λ in stead of $\lambda_{n,a}$ if no confusion happens. $\lambda_{n,a}$ is called an invariant factor of ϕ if

$$\lambda_{n,a} \circ \phi = \lambda_{n,a}. \tag{1}$$

Now let

$$G_{n,a} = \{\phi;\ \phi \in (\mathbf{R}[x_1, \ldots, x_n])^n,\ \lambda_{n,a} \circ \phi = \lambda_{n,a}\},$$

that is, $G_{n,a}$ is the set of polynomial maps of which invariant factor is $\lambda_{n,a}$. The main aim of this note is to determine the structure of $G_{n,a}$.

Let $\Omega_{n,a} = \{x \in \mathbf{R};\ \lambda_{n,a}(x) = 0\}$. Then by the equality (1), for any $n \in \mathbf{N}$,

$$\phi^n(\Omega_{n,a}) \subset \Omega_{n,a},$$

that is, $\Omega_{n,a}$ is an invariant variety of ϕ^n, where ϕ^n denotes n–th iteration of ϕ (see [3]). By using this property, we may investigate the asymptotic dynamical behaviours of iterations of ϕ ([1, 2, 3]). We are led naturally to study the structure of $G_{n,a}$. In fact, we will prove first that $G_{n,a}$ is a group, then we will determine the generators of the group.

In the case $n = 3$, we have showed the following:

Theorem 1 ([2]). *With the notations above, $G_{3,1} = \langle \tau_1, \tau_2, \tau_3, \tau_4 \rangle$ is a group generated by $\tau_1(x, y, z) = (y, x, z)$, $\tau_2(x, y, z) = (z, y, x)$, $\tau_3(x, y, z) = (-x, -y, z)$, $\tau_4(x, y, z) = (x, y, xy - z)$.*