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UNKNOTTING AND FUSION NUMBERS OF RIBBON 2-KNOTS

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Introduction

Hosokawa and Kawauchi [6] proved that any 2-knot can give an unknotted surface after adding enough 1-handles in an appropriate manner. Hosokawa, Maeda, and Suzuki [7] then defined the *unknotting number* $u(K)$ of a 2-knot K to be the least number of such 1-handles.

A ribbon 2-knot is obtained from a trivial $(n+1)$-component 2-link by adding n 1-handles for some n. The *fusion number* $f(K)$ of a ribbon 2-knot K is the least number of n possible for K.

Let K be a ribbon 2-knot. Then Miyazaki [17, Lemma 1] proved:

Proposition 1.

$$u(K) \leq f(K).$$

Note that the fusion number is called the ribbon number in [17]. Let K^* denote the spun 2-knot of a 1-knot K. Let $T_{p,q}$ denote the torus knot of type (p,q). Miyazaki [17, Claim, Remark 2] also showed:

$$u(T^*_{2,2m-1}\#T^*_{2,2m+1}) = 1, \qquad f(T^*_{2,2m-1}\#T^*_{2,2m+1}) = 2, \tag{1}$$

if $m \geq 2$. The purpose of this paper is to provide more examples of ribbon 2-knots that do not satisfy the equality in Proposition 1. First, we will prove:

Theorem 1. *For a nontrivial spun torus knot $T^*_{p,q}$ with $1 < p < q$, we have*

$$u(T^*_{p,q}) = 1, \quad f(T^*_{p,q}) = p-1.$$

Using the composition of a spun torus knot and some copies of a ribbon 2-knot with fusion number one, we will show:

Theorem 2. *For any integers m and n with $0 < m \leq n$, there exists a ribbon 2-knot K such that $u(K) = m$ and $f(K) = n$.*