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ON CLASSIFICATION OF HEEGAARD SPLITTINGS

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1. Introduction

In [12], the Heegaard splittings of orientable Seifert fibred spaces are completely classified. The hyperbolic case, however, is left open, and even in very simple cases, very little is known about the Heegaard structure. A theorem of Moriah and Rubenstein, [11], demonstrates that the Heegaard structure of manifolds obtained by surgery on a cusped hyperbolic manifold are obtained in a natural way from the original cusped manifold's Heegaard splittings. However, this leaves the question of what those Heegaard splittings actually are wide open, as nothing is yet known about the Heegaard splittings of the cusped manifolds.

In this paper, we begin an investigation of Heegaard splittings of manifolds with either zero or one boundary component which possess an *idealized polyhedral decomposition*, or *IPD*, such as exists for cusped hyperbolic manifolds or hyperbolic manifolds with geodesic boundary, by proving the following theorem:

Theorem 1.1. *Let M be a manifold with a single boundary component, and S be an irreducible genus g Heegaard surface for M which is* rigid *with respect to an idealized polyhedral decomposition T. Then one of the compression bodies (in the Heegaard splitting) is a regular neighborhood of some subset of the* 1*-skeleton of T.*

The rigidity condition, together with other ideas necessary in the proof, is defined in Section 2. Section 3 is devoted to the proof of Theorem 1.1.

We note a use for this theorem in Section 4, in which we prove the following corollary:

Corollary 1.2. *Let M be a* 3*-manifold with* 1 *boundary component whose non-rigid (with respect to T) Heegaard splittings are weakly reducible. Assume further that any closed incompressible* 2*-sided surface in M is boundary parallel. Then all Heegaard splittings of M are either induced by T as per* Theorem 1.0, *or are the* amalgamation *of such Heegaard splittings with a trivial Heegaard splitting of a collar of the boundary.*

This follows ideas of [14], in which Gabai's concept of thin position and