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THE CASSON-WALKER INVARIANT FOR BRANCHED CYCLIC COVERS OF S^3 BRANCHED OVER A DOUBLED KNOT

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0. Introduction

In 1985, A. Casson defined an invariant λ for oriented integral homology 3-spheres by using representations from their fundamental group into $SU(2)$ [1]. It was extended to an invariant for rational homology 3-spheres by K. Walker [11]. In 1993, C. Lescop [9] gave a formula to calculate this invariant for rational homology 3-spheres when they are presented by framed links and showed that it naturally extends to an invariant for all 3-manifolds.

Let L be a link in S^3 and let Σ_L^n be its n-fold cyclic branched cover. Define $\lambda_n(L)=\lambda(\Sigma_L^n)$. Then λ_n becomes an invariant of links. For doubles of knots, torus knots and iterated torus knots, A. Davidow (see [3], [4]) calculated Casson's integer invariant for n-fold branched covers, when Σ_K^n is an integral homology sphere. For any links, D. Mullins [10] have succeeded in calculating Casson-Walker's rational valued invariant for 2-fold branched covers, when Σ_L^2 is a rational homology sphere.

In this paper, using C. Lescop's formula and the result of D. Mullins, we will calculate the Casson-Walker invariant for branched cyclic covers of S^3 branched over the m-twisted double of a knot. We will show the following theorem and corollary.

Theorem 3.1. *Let K be a knot in S^3 and D_mK its m-twisted double. Then $\lambda_n(D_mK)$ is determined by $d/dtV_{D_mK}(-1)$ and m where $d/dtV_{D_mK}(-1)$ is the derivative of the Jones polynomial of D_mK at $t=-1$.*

Corollary 3.2. *$\lambda_n(D_mK)$ is determined by $a_1(K)$ and m where $a_1(K)$ is the coefficient of z^2 of the Conway polynomial of K.*

1. Preliminaries

DEFINITION 1.1. The Conway polynomial $\nabla_L(z)$ of an oriented link L is defined by

1. $\nabla_U(z)=1$, where U is an unknot,