Orevkov, S.Y.
Osaka J. Math.
34 (1997), 457–480

ACYCLIC ALGEBRAIC SURFACES BOUNDED BY SEIFERT SPHERES

S. YU. OREVKOV

(Received November 13, 1995)

Let Y be a complex algebraic surface. We say that it is $\boldsymbol{Z}$*-acyclic* (respectively $\boldsymbol{Q}$*-acyclic*) if its reduced homology with coefficients in $\boldsymbol{Z}$ (resp. in $\boldsymbol{Q}$) vanishes. Topologically one can represent Y as a compact 4-manifold with boundary (denote the boundary by S), attached by a collar $S \times [0, 1)$. Call S *the boundary of* Y. If Y is an affine surface in C^m then S is the intersection of Y with a sufficiently large sphere. We say that Y is *A-acyclic at infinity* If S is an A-homology 3-sphere. $(A = \boldsymbol{Z}, \boldsymbol{Q})$. If Y is A-acyclic then it is A-acyclic at infinity. If Y is $\boldsymbol{Q}$-acyclic and $\boldsymbol{Z}$-acyclic at infinity, then it is $\boldsymbol{Z}$-acyclic.

In the paper [18] Ramanujam proved that the only $\boldsymbol{Z}$-acyclic surface bounded by a homotopy 3-sphere is $\boldsymbol{C}^2$, and he also constructed there the first example of a non-trivial $\boldsymbol{Z}$-acyclic (and even contractible) surface. Later on Gurjar and Shastri [7] proved that all $\boldsymbol{Z}$-acyclic surfaces are rationnal. Tom Dieck and Petri [1] classifind all acyclic surfaces which rise out of line configurations on $\boldsymbol{P}^2$. Fujita [5] (resp. Miyanishi, Tsunoda [11] and Gurjar, Miyanishi [6]) classified acyclic surfaces with $\overline{\kappa} = 0$ (resp. $-\infty$ and 1), where $\overline{\kappa}$ denotes the log-Kodaira dimension. Zaidenberg [21] pointed out the connection of $\boldsymbol{Z}$-acyclic surfaces with exotic algebraic and analytic structures on $\boldsymbol{C}^n$, $n \geq 3$. Flenner and Zaidenberg [4] studied deformations of acyclic surfaces.

A *Seifert fibration* (see [19], [17]) on a smooth compact 3-manifold M is a mapping onto a 2-manifold $\pi : M \to B$, which is a locally trivial fibration with fiber S^1 over $B - \{p_1, \ldots, p_r\}$ and which looks near p_j like $D^2 \times S^1 \to D^2$, $(z_1, z_2) \mapsto z_1^{u_j}/z_2^{v_j}$, where $D^2 = \{|z|^2 < 1\} \subset \boldsymbol{C}$, $S^1 = \partial D^2$ and μ_j, ν_j are coprime integers, $\mu_j \geq 2$. The $\pi^{-1}(p_j)$ are called *multiple fibers*; M is called *Seifert manifold* if it admits a Seifert fibration. *Seifert A-homology sphere* (A stands for $\boldsymbol{Z}$ or $\boldsymbol{Q}$) is a Seifert manifold M with $H_*(M; A) = H_*(S^3; A)$. In this case the base B is a 2-sphere. The question, when a Seifert homology sphere bounds an acyclic 4-manifold, was studied, for instance, in [3], [15].

Our main result is:

Theorem 1. *Let* Y *be a smooth algebraic* $\boldsymbol{Q}$*-acyclic surface of logarithmic Kodaira dimension* 2, *bounded by a Seifert* $\boldsymbol{Q}$*-homology sphere with r multiple fibers.*