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ON RATIONALITY OF LOGARITHMIC $\mathbb{Q}$-HOMOLOGY PLANES-I

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1. Introduction

Let V be a normal surface defined over $\mathbb{C}$. Following [3], we say V is *logarithmic* if all its singularities are of quotient type. It is called a $\mathbb{Q}$-*homology plane* if its reduced homology groups with rational coefficients all vanish. Let $\sum = \{p_1, \dots, p_r\}$ denote the set of singularities of V. Then recall that the logarithmic Kodaira dimension of V is defined to be the logarithmic Kodaira dimension of $V \setminus \sum$. In a sequence of three articles beginning with this, we propose to probe the following questions:

Question A: Are all logarithmic $\mathbb{Q}$-homology planes rational?

All logarithmic $\mathbb{Q}$-homology planes with logarithmic Kodaira dimension ≤ 1 are known the be rational (see [3], [2], [7]). Therefore Question A immediately reduces to:

Question B: Are all logarithmic $\mathbb{Q}$-homology planes of logarithmic Kodaira dimension 2 rational?

It may be recalled that in [4], it is proved that all smooth $\boldsymbol{Z}$-homology planes are rational. Adopting the style therein, we can pose the following:

Question C: Let X be a smooth projective surface defined over $\mathbb{C}$. Suppose there is a reduced effective divisor Δ on X such that

i) the irreducible components of Δ generate the $Pic(X) \otimes \mathbb{Q}$;
ii) each connected component of Δ is simply connected;
iii) $\kappa(X, K + \Delta) = 2$.

Then is X a rational surface?

Observe that by blowing up points inside Δ, if necessary, we can assume that Δ is a normal crossing curve. By blowing down, if necessary, we can assume that

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