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Osaka J. Math.
34 (1997), 357–361

SELF DUAL GROUPS OF ORDER p^5 (p AN ODD PRIME)

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(Received December 14, 1995)

1. Introduction

Let G be a finite group, $\mathrm{Irr}(G) = \{\chi_1, \cdots, \chi_k\}$ be the set of all irreducible characters, $\mathrm{Cl}(G) = \{C_1, \cdots, C_k\}$ be the conjugacy classes of G, and x_i be a representative of C_i. We call G self dual if (by renumbering indices)

(*) $|C_j|\chi_i(x_j)/\chi_i(1) = \chi_j(1)\chi_j(x_i)$, for all i, j.

This condition is found in E. Bannai [1]. T. Okuyama [4] proved that self dual groups are nilpotent, and that a nilpotent group is self dual if and only if its all Sylow subgroups are self dual. So if we consider self dual groups we may deal with only p-groups. Obviously abelian groups are self dual. Some examples of self dual groups are discussed in [2].

If G is self dual it is easy to check that $|C_i| = \chi_i(1)^2$ for all i. It is easy to see that non abelian p-groups of order at most p^4 cannot satisfy this condition, and so they are not self dual. By the classification of groups of order 2^5, there is no group of order 2^5 satisfying this condition. For odd p, in classification table of groups of order p^5 [3], we can see that one isoclinism family Φ_6 satisfies this condition. We will show that all of groups in Φ_6 are self dual.

2. Definition of groups

We fix an odd prime p. Let G be a p-group of order p^5 which belongs to Φ_6 defined in [3], namely

$$G = \langle a_1,\ a_2,\ b,\ c_1,\ c_2 \mid [a_1, a_2] = b,\ [a_i, b] = c_i,\ a_i^p = \zeta_i,\ b^p = c_i^p = 1\ (i = 1,\ 2)\rangle,$$

where (ζ_1, ζ_2) is one of the followings:

(1) (c_1, c_2),
(2) (c_1^k, c_2), where $k = g^r$, $r = 1, 2, \cdots, (p-1)/2$,
(3) $(c_2^{-r/4}, c_1^r c_2^r)$, where $r = 1$ or ν,
(4) (c_2, c_1^ν),
(5) $(c_2^k, c_1 c_2)$, where $4k = g^{2r+1} - 1$, $r = 1, 2, \cdots, (p-1)/2$,
(6) $(c_1, 1)$, $p > 3$,
(7) $(1, c_1^r)$, where $r = 1$ or ν, and $p > 3$,
(8) $(1, 1)$,