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GROUPS WITH SOME COMBINATORIAL PROPERTIES

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1. Introduction

In [1], E. Bannai introduced the concept of fusion algebras at an algebraic level, a purely algebraic concept for fusion algebras in mathematical physics. He showed that there exists a one-to-one correspondence between character algebras (Bose-Mesner algebras at algebraic level) and fusion algebras at an algebraic level. The concept of character algebras is a purely algebraic concept for Bose-Mesner algebras of association schemes.

For any commutative association scheme, a character algebra and the corresponding fusion algebra at algebraic level are constructed. But this fusion algebra at an algebraic level is far from a fusion algebra in mathematical physics. A fusion algebra in mathematical physics is integral, its matrix S is symmetric (and unitary), and it has the modular invariance property. But these are not true for fusion algebras at an algebraic level. So he asked which fusion algebra at an algebraic level have these properties.

In this paper, we construct some p-groups and check the properties of their group association schemes. For our groups, the fusion algebras are integral and S is unitary but not necessary symmetric. Section 4 is a generalization of [2].

2. Fusion algebras at an algebraic level and character algebras

For the definitions of fusion algebras and character algebras, we refer to [1, Definition 1.1 and 2.5].

Theorem 2.1 [1, Theorem 3.1]. *There exists a natural one-to-one correspondence between fusion algebras at an algebraic level and character algebras.*

The correspondence in Theorem 2.1 is the following. Let $\tilde{\mathfrak{A}} = \langle y_0, y_1, \cdots, y_d \rangle$ be a character algebra with basis $y_0, y_1, \cdots, y_d$ and the multiplication

$$y_i y_j = \sum_{k=0}^{d} p_{ij}^k y_k.$$