Kikuchi, K. and Negoro, A.
Osaka J. Math.
34 (1997), 319–335

ON MARKOV PROCESS GENERATED BY PSEUDODIFFERENTIAL OPERATOR OF VARIABLE ORDER

KOJI KIKUCHI and AKIRA NEGORO

(Recieved February 2, 1996)

1. Introduction

The relationship between diffusion processes and differential equations has been studied with great details. However the relationship between pure jump type Markov processes and evolution equations has not been studied in full details.

Recently, N. Jacob and H. Leopld [2] have shown that there exists a Feller semigroup generated by the pseudodifferential operator whose symbol is the function $-\langle\xi\rangle^{\sigma(x)} = -(\sqrt{1+|\xi|^2})^{\sigma(x)}$, which we denote $-\langle D_x\rangle^{\sigma(X)}$. Here the order function $\sigma(x)$ is the sum of some function in Schwartz class and some constant and satisfies $0 < \inf\sigma \leq \sup\sigma \leq 2$. For this purpose they have introduced a suitable function space. This space is a kind of Sobolev space of variable order. Here $\langle\xi\rangle^{\sigma(x)}$ have been used as a weight function. They have shown that the restriction of $\langle D_x\rangle^{\sigma(X)}$ to this Sobolev space satisfies the conditions in Hille-Yosida-Ray theorem. But they do not give conditions for that a general pseudodifferential operator generates a Feller semigroup. In this paper we give one answer to this problem.

We want to inverstigate the relationship between Markov processes and evolution equations with respect to pseudodifferential operators by developing their ideas. In order to complete our theory we should restrict functions which we treat to $H^{-\infty}$ instead of $\mathcal{S}'(\boldsymbol{R}^d)$ at first of all. And considering the function $\langle\xi\rangle$ ($\xi \in \boldsymbol{R}^d$) as standard weight function, we define the Sobolev space $H^{\sigma(\cdot)}(\boldsymbol{R}^d)$ with variable order by the same way as in the definition of such spaces with constant order, where the order function σ is in $\mathcal{B}^\infty(\boldsymbol{R}^d)$. This definition which is slightly different from one in [2] allows us to show that if σ_1 and σ_2 are in $\mathcal{B}^\infty(\boldsymbol{R}^d)$ and satisfy $\sigma_1(x) \leq \sigma_2(x)$ for any $x \in \boldsymbol{R}^d$, then $H^{\sigma_2(\cdot)}(\boldsymbol{R}^d) \subset H^{\sigma_1(\cdot)}(\boldsymbol{R}^d)$, and if P is an elliptic pseudodifferential operator in the class $\mathcal{S}^{\sigma}_{\rho,\delta}(0 \leq \delta < \rho \leq 1)$, then the space $\{u \in H^{-\infty}; Pu \in L^2(\boldsymbol{R}^d)\}$ coincides with $H^{\sigma(\cdot)}(\boldsymbol{R}^d)$. Combining the ideas in N. Jacob and H. Leopld [2] with these facts, we obtain that there exists a Feller semigroup $\{T_t\}_{t\geq 0}$ generated by a storngly elliptic pseudodifferential operator P with suitable variable order. Using the method proposed in [6], we see that, for $u_0 \in C_0^\infty(\boldsymbol{R}^d)$, $u = T_t u_0$ is a unique solution to the initial-value problem $\partial_t u - Pu = 0$, $u(0) = u_0$ and also we can construct its fundamental solution which