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INFINITESIMAL GENERATORS OF NONHOMOGENEOUS CONVOLUTION SEMIGROUPS ON LIE GROUPS

Dedicated to Professor Masatoshi Fukushima on the occasion of his 60th birthday

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1. Introduction

In 1956, Hunt [3] characterized all possible homogeneous convolution semigroups of probability distributions on a Lie group through the representations of their infinitesimal generators. Let $\{\mu_t\}_{t>0}$ be a convolution semigroup of probability distributions defined on a Lie group G of dimension d. It defines a semigroup of linear operators $\{T_t\}_{t>0}$ on $\mathscr{C}$ by setting $T_tf(\sigma)=\int f(\sigma\tau)\mu_t(d\tau)$, where $\mathscr{C}$ is the Banach space consisting of bounded continuous functions f on G (such that $\lim_{\sigma\to\infty}f(\sigma)$ exists if G is noncompact). Then the domain $\mathscr{D}(A)$ of its infinitesimal generator A contains $\mathscr{C}_2$ (a space consisting of $\mathscr{C}_2$-functions on G) and Af, $f\in\mathscr{C}_2$ is represented by

$$(1.1)\qquad Af(\sigma)=\frac{1}{2}\sum_{i,j}a^{ij}X_iX_jf(\sigma)+\sum_i b^iX_if(\sigma)$$
$$+\int_G\Big(f(\sigma\tau)-f(\sigma)-\sum_i x^i(\tau)X_if(\sigma)\Big)\nu(d\tau).$$

Here $X_1,\cdots,X_d$ constitute a basis of the Lie algebra of G regarding them as left invariant first order differential operators (vector fields), $A=(a^{ij})$ is a symmetric nonnegative definite matrices, $b=(b^i)$ is a vector and ν is a measure on G such that $\nu(\{e\})=0$ and $\int\phi(\sigma)\nu(d\sigma)<\infty$, where e is the unit element of G. Further, $x^1,\cdots,x^d$, ϕ are $\mathscr{C}_2$-functions on G satisfying (3.1) and (3.2). Conversely the above operator determines a unique convolution semigroup.

In this paper we study nonhomogeneous convolution semigroups $\{\mu_{s,t}\}_{0<s<t<\infty}$ of probability distributions on a Lie group. In the first part (Sections 2–4), we characterize them by representing their infinitesimal generators $A(t), t>0$, similarly as (1.1), where the triple (A,b,ν) in the representation on $A(t)$ depends on t. We remark that a similar representation of the infinitesimal generator has been obtained by Maksimov [7] in the case where the underlying Lie group is compact. However,