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QUASIREGULAR MAPPINGS AND *d*-THINNESS

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0. Introduction

In the study of boundary behavior of solutions of the classical Dirichlet problem thinness is an important notion. Recently notion of p-thinness (or p-thickness) is introduced in nonlinear potential theory and studied deeply. For $p=2$, p-thinness (resp. p-thickness) coinsides with thinness (resp. thickness) with respect to the classical potential theory. In this note we are especially concerned with d-thinness (or d-thickness) on the d dimensional Euclidean space $\boldsymbol{R}^d$ $(d \geq 2)$. The purpose of this note is to consider whether d-thinness (or d-thickness) is quasiregularly invariant or not. We obtain

Theorem 0.1. *Let G be a subdomain of $\boldsymbol{R}^d$ $(d \geq 2)$, E a subset of G, ξ a point of $G \backslash E$, and f a quasiregular mapping from G into $\boldsymbol{R}^d$. If E is d-thick at ξ, then $f(E)$ is d-thick at $f(\xi)$.*

The following theorem is an immediate conclusion of our main Theorem 0.1.

Theorem 0.2 (O. Martio and J. Sarvas [11]). *Let G be a subdomain of $\boldsymbol{R}^d$ $(d \geq 2)$, E a subset of G, ξ a point of $G \backslash E$ and f a quasiconformal mapping from G into $\boldsymbol{R}^d$. If E is d-thin (resp. d-thick) at ξ, then $f(E)$ is d-thin (resp. d-thick) at $f(\xi)$.*

Theorem 0.1 is obtained by the results of nonlinear potential theory. For $d=2$, H. Shiga [15] also obtains Theorem 0.2 by a different method from [11].

This note is organized as follows. In §1 we give preliminaries and discuss whether d-thickness (or d-thinness) is invariant by quasiregular mappings (see Theorem 1.1). In §2 and §3 we are concerned with applications of Theorem 0.2. In §2, comparison between thinness and minimal thinness, and Theorem 0.2 give us the quasiconformal invariance of minimal thinness under a condition (see Theorem 2.1). In §3 we prove that the harmonic dimension of Heins' covering surface is quasiconformally invariant under a condition (see Theorem 3.1).

1. Preliminaries and Proof of Theorem 0.1