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ON CONTIGUITY RELATIONS OF JACKSON'S BASIC HYPERGEOMETRIC SERIES Υ_1 (*a*; *b*; *c*; *x*, *y*, 1/2) AND ITS GENERALIZATIONS

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1. Introduction. Our object is the following q-hypergeometric series of confluent type

$$(1)\qquad \sum_{\nu_1,\cdots,\nu_m=1}^{\infty}\frac{(\alpha:\Sigma_{i=1}^m\nu_i)_q(\beta_1:\nu_1)_q\cdots(\beta_{m-1}:\nu_{m-1})_q}{(\gamma:\Sigma_{i=1}^m\nu_i)_q(1:\nu_1)_q\cdots(1:\nu_{m-1})_q(1:\nu_m)_q}y_1^{\nu_1}\cdots y_m^{\nu_m}q^{\nu_m(\nu_m-1)/2},$$

where q is a complex number satisfying $|q|<1$. We have used the following notation $(a:n)_q=(a)_q(a+1)_q\cdots(a+n-1)_q$, $(a)_q=\dfrac{1-q^a}{1-q}$. When $m=1$, this series gives a q-analog of Kummer's hypergeometric series. This series (1) coincides with Jackson's basic double hypergeometric series $\Upsilon_1(\alpha;\beta_1;\gamma;y_1,y_2,1/2)$ [7] when $m=2$. Two series of this form are said to be contiguous if parameters α, β_i, γ and α', β'_i, γ' corresponding to them differ at most 1 for each pair. We also say that two such series are contiguous to each other. For later convenience we introduce new parameters

$$(2)\qquad \alpha=\mu_2+1,\ \gamma=\mu_2+\mu_3+2,\ \beta_i=-\mu_i\ (4\le i<n),\ \sum_{i=1}^{n-1}\mu_i=-2.$$

We also rename independent variables as $y_i=x_{i+3}$ and set $n=m+3$ to make formulas appear later simple. In these new variables and parameters the series (1) looks as

$$(3)\qquad \sum_{\nu_4,\cdots,\nu_n=1}^{\infty}\frac{(\mu_2+1:\Sigma_{i=4}^n\nu_i)_q(-\mu_4:\nu_4)_q\cdots(-\mu_{n-1}:\nu_{n-1})_q}{(\mu_2+\mu_3+2:\Sigma_{i=4}^n\nu_i)_q(1:\nu_4)_q\cdots(1:\nu_{n-1})_q(1:\nu_n)_q}x_4^{\nu_4}\cdots x_n^{\nu_n}q^{\nu_n(\nu_n-1)/2}.$$

We shall describe q-difference operators which increase one of the μ_is and decrease one of the μ_is. We call such operators raising and/or lowering operators.

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