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THE VORTEX FILAMENT EQUATION AND A SEMILINEAR SCHRÖDINGER EQUATION IN A HERMITIAN SYMMETRIC SPACE

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0. Introduction

We consider the initial value problem of the vortex filament equation on $\boldsymbol{R}^3$:

$$\gamma_t = \gamma_x \times \gamma_{xx} \quad (|\gamma_x| \equiv 1) \qquad (\times \text{ is the exterior product}).$$

In this paper, we will prove the existence and the uniqueness of a classical solution for the initial value problem, and generalize it to the case of curves in 3-dimensional space forms. We also consider related semilinear Schrödinger equations for curves in Kähler manifolds. It is remarkable that we need symmetric spaces as manifolds for infinite time existence of solutions.

More precisely, we will get the following results.

Theorem 1.5. *The initial value problem* $\gamma_t = \gamma_x \times \gamma_{xx}$ $(|\gamma_x| \equiv 1)$ *for closed curves in the euclidean space* $\boldsymbol{R}^3$ *has a unique solution on* $-\infty < t < \infty$.

Theorem 2.2. *Let* M *be an oriented* 3*-dimensional riemannian manifold with constant curvature* c. *The initial value problem* $\gamma_t = \gamma_x \times \nabla_x \gamma_x$ $(|\gamma_x| \equiv 1)$ *for closed curves in* M *has a unique solution on* $-\infty < t < \infty$ *for any initial data.*

Theorem 3.5. *Let* M *be a Kähler manifold. The initial value problem* $\xi_t = J\nabla_x \xi_x$ *for closed curves in* M *has a unique short time solution for any initial data.*

Theorem 4.2. *Let* M *be a complete locally hermitian symmetric space. The initial value problem* $\xi_t = J\nabla_x \xi_x$ *for closed curves has a unique all time solution* $(-\infty < t < \infty)$ *for any initial value.*

We also get

Theorem 1.3. *Hasimoto's transformation is well defined, even when the curvature vanishes at some point.*