Gotoh, T.
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COMPACT MINIMAL CR-SUBMANIFOLDS WITH THE LEAST NULLITY IN A COMPLEX PROJECTIVE SPACE

TOHRU GOTOH

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1. Introduction

A minimal submanifold M of a Riemannian manifold $\tilde{M}$ is nothing but a critical point of the volume functional induced from the Riemannian measure of $\tilde{M}$. The first variational formula then states, roughly speaking, that the gradient of the volume functional is the negative of the mean curvature vector field of M. Concerning with the second variational formula for the volume functional, the nullity and the index play an important role in the theory of minimal submanifolds. Geometrically, the nullity means how many deformations of M preserving the volume there are, while the index means how many deformations of M decreasing the volume there are. For precise definition, see Section 2.

In this paper we investigate the following problem: estimate the nullity from below and further determine minimal submanifolds with the least nullity. The first results for this problem are found in [12], where Simons has proved the following (cf. Exmple 3.6): the nullity of a compact minimal submanifold M in the Euclidean sphere is bounded from below by $(\dim M+1)\operatorname{codim} M$, and furthermore it attains the lower bound if and only if M is totally geodesic. The method that Simons used for Euclidean sphere will be explained in more general form in Section 3. Next Kimura [6] and Ohnita [10] have obtained respectively results for complex submanifolds and for totally real submanifolds in a complex projective space (cf. Example 3.8). On the other hand, Ohnita [10] has computed the nullities of totally geodesic submanifolds in compact symmetric spaces of rank one. Especially he obtained that the nullity of every compact totally geodesic submanifold in a compact rank one symmetric space is equal to its Killing nullity. Recently the present author [4] has obtained a result for real hypersurfaces in a complex projective space (cf. Example 3.7, Fact 7.13).

The purpose in this paper is to investigate the problem for some wider class of submanifolds in a complex projective space, that is, the class of minimal CR-submanifolds. The following theorem summarizes the main results (Theorem 5.9, 5.10, 7.6);