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SEIBERG-WITTEN INVARIANTS ON NON-SYMPLECTIC 4-MANIFOLDS

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Let X be an oriented, closed Riemannian 4-manifold. There is an integral cohomology class which reduces mod (2) to the second Stieffel-Whiney class $w_2(X)$. This integral cohomology class induces a $Spin^c$-structure on X. Seiberg and Witten in [10] introduced a new invariant on X which is a differential-topological invariant. Taubes in [9] proved that every closed symplectic 4-manifold has a non-trivial Seiberg-Witten invariant. The Seiberg-Witten invariants of connected sums of 4-manifolds with $b_2^+>0$ identically vanish. Kotschick, Morgan and Taubes in [8] showed that there are closed oriented 4-manifolds with nontrivial Seiberg-Witten invariants which do not admit symplectic structures. They considered the case which is the first Betti number $b_1(N)=0$. We would like to generalize their theorem by giving a certain condition instead of $b_1(N)=0$, of course our case will cover their case. We introduce their theorem:

Theorem ([8]). *Let X be a manifold with a nontrivial Seiberg-Witten invariant with $b_2^+(X)>1$, and let N be a manifold with $b_1(N)=b_2^+(N)=0$ whose fundamental group has a nontrivial finite quotient. Then $M=X\#N$ has a non-trivial Seiberg-Witten invariant but does not admit any symplectic structure.*

Let M be a closed symplectic 4-manifold and let $M=X\#N$ be a smooth connected sum decomposition. By the vanishing theorem of Seiberg-Witten invariants and non-trivial Seiberg-Witten invariants for symplectic manifolds, one of the summands, say it N, has a negative definite intersection form. By Donaldson's Theorem [5] there is a basis $\{e_1,\cdots,e_n\}$ of the free part of $H^2(N,\boldsymbol{Z})$ such that in this basis the intersection form of N is diagonal, where n is the rank of $H^2(N,\boldsymbol{Z})$. An element $\alpha\in H^2(N,\boldsymbol{Z})$ is said to be characteristic if the intersection number $\alpha\cdot x=x\cdot x$ mod (2) for any $x\in H^2(N,\boldsymbol{Z})$. If α is characteristic, then $\alpha\equiv w_2(N)$ modulo 2.

Lemma 1. *Let N be a closed oriented Riemannian 4-manifold with $b_2^+(N)=0$ and let $\{e_1,\cdots,e_n\}$ is a basis for the free part of $H^2(N,\boldsymbol{Z})$ such that $e_i\cdot e_j=-\delta_{ij}$.*

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