

SPECIAL GENERIC MAPS AND L^2 -BETTI NUMBERS

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1. Introduction

A smooth map between manifolds with only definite fold singular points is called a *special generic map*. Bulet and de Rham [2] first defined a special generic map and showed that a closed 3-manifold admits a special generic map into \mathbf{R}^2 if and only if it is diffeomorphic to S^3 or the connected sum of some S^2 -bundles over S^1 . Furthermore, Saeki determined completely those closed manifolds which admit special generic maps into \mathbf{R}^2 in [20]. Further results are known about the topology of manifolds which admit special generic maps (see [12], [19], [20], [21], [22], [23]). In this paper we study special generic maps of closed manifolds into open manifolds and their singular sets by using L^2 -Betti numbers which were introduced by Atiyah [1].

Theorem 1.1. *Let M be a closed connected n -dimensional manifold such that the q -th L^2 -Betti number $b_q^{(2)}(M)$ of M is not zero for some $q \leq n/2$. Then for any open p -dimensional manifold N with $p \leq q$, M does not admit a special generic map into N .*

By using this theorem, we see that no closed hyperbolic manifold of dimension $2n$ admits a special generic map into an open p -dimensional manifold N with $p \leq n$ (see Corollary 5.2).

As is seen in [2], the topology of the singular set of a special generic map is not determined only by the topology of the source manifold. However some results are known about the relationship between the topology of the singular set of a special generic map and that of the source manifold (see [20], [22], [23]). We study the order of the fundamental group of the singular set by using the L^2 -Betti numbers of the source manifold. In order to state the next theorem, we recall the definition of residually finite groups. A group G is called *residually finite* if every non-trivial element of G is mapped nontrivially in some finite quotient group of G by a homomorphism.

Theorem 1.2. *Let M be a closed connected n -dimensional manifold such that $\pi_1(M)$ has infinite order and let $f : M \rightarrow N$ be a special generic map of M into an open*