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CYCLIC SURGERY ON GENUS ONE KNOTS

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0. Introduction

The real projective 3-space, denoted by RP^3, is identified with the lens space of type (2,1). Then one can ask: when can RP^3 be obtained by Dehn surgery on a knot in the 3-sphere S^3? Clearly RP^3 is obtained by Dehn surgery on a trivial knot. However, it is conjectured that no Dehn surgery on a nontrivial knot K in S^3 yields RP^3 (cf. [1,4]). It is known to be true if K is a composite knot [3], a torus knot [9], an alternating knot [10], a satellite knot [1,12,13], or a symmetric knot [1].

In this paper we prove the conjecture for genus one knots.

Theorem 0.1. *Real projective 3-space RP^3 cannot be obtained by Dehn surgery on a genus one knot in S^3.*

This will be proved by applying the combinatorial techniques developed in [2,5,6,8].

1. Preliminaries

Let K be a genus one knot which is neither a torus knot nor a satellite knot. Let $N(K)$ be a tubular neighborhood of K and let $E(K)=S^3-\operatorname{int}N(K)$. Suppose that some surgery on K yields RP^3, that is, $E(K)\cup J=RP^3$ where J is a solid torus. By [2], the surgery coefficient is ± 2.

Let $P^2\subset RP^3$ be a projective plane which intersects J in a disjoint union of meridian disks of J. We assume that $|P^2\cap J|$ is minimal among all projective planes in RP^3 that intersect J in a family of meridian disks of J. Let $p=|P^2\cap J|$ and $P=P^2\cap E(K)$. Then P is incompressible in $E(K)$ by the minimality of p. If p is even, then $E(K)$ would contain a closed non-orientable surface by attaching tubes to ∂P. Hence p is odd. Furthermore, if $p=1$ then K is either a torus knot or a $(2,\pm 1)$-cable knot. Thus $p\neq 1$.

Let Q be a genus one Seifert surface for K. We may assume that P and Q intersect transversely, and ∂Q intersects each component of ∂P exactly twice. By the incompressibility of P and Q, we can assume that no circle component of