Matignon, D.
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DEHN SURGERY ON A KNOT WITH THREE BRIDGES CANNOT YIELD P^3

DANIEL MATIGNON

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Introduction

We work in the P.L. category, with compact manifolds. Two knots k and k' are equivalen if and only if there exists a homeomorphism $h: S^3 \to S^3$ such that $h(k)=k'$. They are equivalent if and only if their complements S^3-k and S^3-k' are homeomorphic. This is a direct consequence of Theorem 2 (p.371) of [9]: "*nontrivial Dehn surgery on a nontrivial knot never yield* S^3".

Similarly, can the complement of a knot k in S^3 and that of a knot in P^3 be homeomorphic? This question is equivalent to: can a Dehn surgery on a knot k in S^3 yield P^3? C. McA. Gordon has conjectured that this is not possible if k is not trivial (Conjecture 5.6 of [7], p.12).

Trivial surgery always yields S^3 and non-trivial surgeries on a trivial knot yield lens spaces. The $2/1$-surgery along the trivial knot give P^3. From now on, here all knots are nontrivial. In this paper, we shall prove that P^3 cannot be obtained by Dehn surgery on a class of knots.

Theorem 1. *P^3 can not be obtained by Dehn surgery on a non-trivial knot with at most 3 bridges.*

For many kinds of knots the same result is known:

(i) torus knots by L. Moser ([12] 1971);
(ii) satellite knots by C. McA. Gordon ([8] 1990) and the "Cyclic Surgery Theorem" ([2] 1987);
(iii) symmetric knots by S. Bleiler with R. Litherland ([1] 1989) and by S. Wang with Q. Zhou ([17] 1992),
(iv) knots with genus 1 by M. Domergue ([3] 1991),
(v) knots with 2 bridges; this is a consequence of the note by N. Sayari, G. Hoquenghem and myself (in [11] 1995, this is in the thesis of N. Sayari) see also [1] and [16].

But for a knot with 3 bridges the answer remained unknown.

Let k be a knot in S^3, $N(k)$ a regular neighbourhood of k in S^3, and