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DRINFELD MODULAR FUNCTIONS

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0. Introduction

In 1973, Drinfeld introduced the notion of elliptic modules, which is now known as Drinfeld modules. After that the analogies between number fields and function fields have many interesting new aspects. Drinfeld modular function theory is one of these.

Drinfeld modular functions were studied by several mathematicians and known to have many properties analogous to those of classical elliptic functions, such as the generators of modular function fields, Galois groups between them. ([5],[8],[9]).

In the first part of this note, we establish some more properties of Drinfeld modular functions in analogy with those obtained by Shimura. In [12], Shimura proved his exact-sequence and his reciprocity law. Lang proved the exact-sequence another way in [11] using the isogeny theory. Shimura's proof of the reciprocity law is not easy. For example he used the parametrizations of the models of a modular function field over $\boldsymbol{Q}$. In [11], Lang avoided Shimura's method using the decomposition gorups which is well-known in algebraic number theory. In this article, we will follow Shimura's method to prove the exact-sequence, and Lang's method to prove the reciprocity law in the function field case.

In the second part, we go on to study two variable Drinfeld modular functions in analogy with two variable elliptic functions studied by Berndt. ([1]). In [2], he also gerneralized Shimura's exact-sequence and his reciprocity law corresponding to this extended modular function fields. We discuss the analogies of these in the Drinfeld setting.

1. Definitions and basic facts

Let $A=\boldsymbol{F}_q[T]$, $k=\boldsymbol{F}_q(T)$, k_∞ be the completion of k at $\infty=(\frac{1}{T})$, and C the completion of the algebraic closure of k_∞. Then C has an absolute value extending that of k_∞. By an A-lattice in C, we mean a projective A-submodule Λ of C which is discrete in the topology of C. A meromorphic function f on C is said to be *even* if $f(\mu z)=f(z)$ for every $\mu\in\boldsymbol{F}_q^*$. A meromorphic function f on C is

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