1. Introduction

The purpose of this paper is to study the dimension formula for the invariant ring $C[\alpha]_{\Gamma_g}$, which may be considered as the ring of code polynomials in genus 4. We also give all characteristic polynomials of elements in $H_4$. The main ingredient is the determination of the conjugacy classes of the symplectic group $Sp(8,2)$. Our result will be useful for the investigation of the Siegel modular forms in genus four.

We recall from [10, 11] that the finite group $H_g$ is (up to $\pm 1$) just the image of the modular group $\Gamma_g = Sp(2g, Z)$ under the theta representation (of index 1) and that the ring of modular forms of even weight is given by

$$A(\Gamma_g, (2)) = \bigoplus_{2k} [\Gamma_g, \kappa] = (C[\alpha]_{H_g} / \langle \text{relation} \rangle)^N,$$

where $N$ denotes the normalization in its field of fractions and “relation” are the theta relations. However, the generators and the dimension formulas for $A(\Gamma_g)$ are known only for genus $g \leq 3$.

On the other hand, the invariant ring $C[\alpha]_{H_g}$ may be considered as the ring of code polynomials in genus $g$. In [9], there is the definition of the $g$-th weight polynomial for codes (codes mean the binary linear codes) and the connections among codes, lattices, the invariant rings of the finite groups, and the theory of modular forms were studied (cf. [1], [4], [5], [8], [16]). In particular, it was shown that the invariant ring of the group $\langle H_g, \zeta_8 \rangle$, which is the subring of $C[\alpha]_{H_g}$, is generated by the $g$-th weight polynomials for self-dual doubly-even codes, where $\zeta_8$ is the primitive 8-th root of unity. This invariant ring corresponds to the ring of the modular forms of weights divisible by 4.

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