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LEVEL-TWO-STRUCTURES AND HYPERELLIPTIC CURVES

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1. Introduction

The main idea in this paper is to apply the results in ([31],···,[34]) to geometry. The paper contains some results on Specht modules due to several exceptional isomorphisms of subgroups of the symplectic groups to symmetric groups. The geometry of the Igusa desingularization in genus 2 and level 2 is described.

We always choose as coordinates the theta constants of second kind. The ordianry theta constants are used for auxiliary purposes. They give the equations of the Humbert surfaces in genus two or of the hyperelliptic points in genus three.

2. Notations and first results

Throughout the paper we will use the following notations in accordance with [31], [32]. General references are [9], [24], [26], [27], [29] and [37].

$$\boldsymbol{H}_g=\{\tau\in\mathrm{Mat}_{g\times g}(\boldsymbol{C})\mid\tau \textit{ symmetric}, \ \mathrm{Im}(\tau)>0\},$$

$$\Gamma_g=Sp(2g,\boldsymbol{Z}),$$

$$\Gamma_g(n)=\mathrm{Ker}(\Gamma_g\to Sp(2g,\boldsymbol{Z}/n)).$$

For a subgroup Γ of finite index in Γ_g we denote by $A(\Gamma)=\oplus_k[\Gamma,k]$ the ring of modular forms for Γ and by $\mathscr{A}_g(\Gamma)=\mathrm{Proj}(A(\Gamma))$ the corresponding Satake compactification. (This is *not* the standard notation.) The variety $\mathscr{A}_g(\Gamma)$ contains $\boldsymbol{H}_g/\Gamma$ as an open dense subset. $\boldsymbol{H}_g/\Gamma$ is a coarse moduli space for principally polarized abelian varieties (ppav) with level-Γ-structure. The ring $A(\Gamma)$ is a normal graded integral domain finitely generated as an algebra over $\boldsymbol{C}=[\Gamma,0]$. For general facts about such graded rings see [30].

The thetas (of second kind) are given by (we use Mumford's notation f_a)

$$f_a(\tau)=\theta\begin{bmatrix}a\\0\end{bmatrix}(2\tau)=\sum_{x\in\boldsymbol{Z}^g}\exp 2\pi i\left(\tau\left[x+\frac{1}{2}a\right]\right)$$