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ON *QF*-RINGS WITH CYCLIC NAKAYAMA PERMUTATIONS

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0. Introduction

Let R be a basic Quasi-Frobenius ring (in brief, QF-ring) and $E=\{e_1,e_2,\cdots,e_n\}$ be a complete set of orthogonal primitive idempotents of R. For any e in E, there exists a unique f in E such that the top of fR is isomorphic to the bottom of eR and the top of Re is isomorphic to the bottom of Rf. Then the permutation $\begin{pmatrix} e_1 & e_2 & \cdots & e_n \\ f_1 & f_2 & \cdots & f_n \end{pmatrix}$ is said to be a Nakayama permutation of R.

If R is a QF-ring, then R contains a basic QF-subring R^0 such that R is Morita equivalent to R^0. So Nakayama permutations of R^0 are considered and we call these Nakayama permutations of R.

It is well-known that Nakayama permutations of a group algebra of a finite group over a field are identity. This paper is concerned with QF-rings with cyclic Nakayama permutations. Our main result is the following:

Theorem. *If R is a basic QF-ring such that for any idempotent e in R, eRe is a QF-ring with a cyclic Nakayama permutation, then there exist a local QF-ring Q, an element c in the Jacobson radical of Q and a ring automorphism σ of Q for which R is represented as a skew-matrix ring*:

$$R \simeq \begin{pmatrix} Q & \cdots & Q \\ & \cdots & \\ Q & \cdots & Q \end{pmatrix}_{\sigma,c,n}.$$

Throughout this paper R will always denote an associative ring with identity and all R-modules are unitary. The notation M_R (resp. ${}_RM$) is used to denote that M is a right (resp. left) R-module. For a given R-module M, $J(M)$ and $S(M)$ denote its Jacobson radical and socle, respectively. For R-modules M and N, $M \underset{\sim}{\subseteq} N$ means that M is isomorphic to a submodule of N. And, for R-modules M and N, we put $(M,N)=\mathrm{Hom}_R(M,N)$ and in parrticular, we put $(e,f)=(eR,fR)$ $=\mathrm{Hom}_R(eR,fR)$ for idempotents e,f in R.