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GEOMETRY OF PLANE CURVES VIA TSCHIRNHAUSEN RESOLUTION TOWER

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1. Introduction.

The weight vectors of a resolution tower of toric modifications for an irreducible germ of a plane curve C carry enough information to read off invariants such as the Puiseux pairs, multiplicities, etc [29]. However, each step of the inductive construction of a tower of toric modifications depends on a choice of the modification local coordinates. This ambiguity makes it difficult to study the equi-singularity problem of a family of germs of plane curves or to study a global curve. It is the purpose of this paper to make a canonical choice of the modification local coordinates (u_i, v_i) (Theorem 4.5), and to obtain a canonical sequence of germs of curves $\{C_i; i=1,\cdots,k\}$ $(C_k=C)$ such that the local knot of the curve C_i is a compound torus knot around the local knot of the curve C_{i-1}. We will show that the local equations $h_i(x,y)$ of the the germs $\{C_i; i=1,\cdots,k\}$ are the Tschirnhausen approximate polynomials of the local equation $f(x,y)$ for C, provided that $f(x,y)$ is a monic polynomial in y.

The importance of the Tschirnhausen approximate polynomials was first observed by Abhyankar-Moh [3,4], and our work is very much influenced by them. However, our result gives not only a geometric interpretation of [3,4] but also a new method to study the equi-singularity problem, see [35], for a given family of germs of irreducible plane curves $f(x,y,t)=0$ whose Tschirnhausen approximate polynomials $h_i(x,y)$, $i=1,\cdots,k-1$ do not depend on t.

In section 6, we show that a family of germs of plane curves $\{f_t(x,y)=0\}$ with Tschirnhausen approximate polynomials $h_i(x,y)$, $i=1,\cdots,k-1$ not depending upon t and satisfying an additional intersection condition is equi-singular (Theorem 6.2). In section 8, we will give a new proof and a generalization of the Abhyankar-Moh-Suzuki theorem from the viewpoint of the equi-singularity at infinity (Theorems 8.2, 8.3, 8.7).

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