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ON ZARISKI'S PROBLEM CONCERNING THE 14TH PROBLEM OF HILBERT

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0. Introduction

Zariski proposed the following problem while he was trying to solve the 14th Problem of Hilbert ([1]):

Let A be a normal affine ring over a field k and let L be a function field over k such that L is a subfield of the field of fractions of A. Is then $A \cap L$ an afine ring over k?

The writer discussed the problem introducing a new method to construct a ring defined by an ideal I of an integral domain R ([2]). Namely, letting K be the field of fractions of R, we defined the I-transform of R to be the ring $\{x \in K | xI^n \subseteq R$ for some $n \in \boldsymbol{N}\}$. He discussed the I-transform of R also in [3].

These articles [2], [3] were written, dreaming an affirmative answer of the 14th Problem of Hilbert. But, we know already that the problem has a negative answer, and the writer wishes to write down the main results of articles [2] and [3], without such a dream and in a generalized form.

We begin with some preliminaries on Krull rings and on discrete valuation rings. Then, we give some characterization of rings which are obtained as the intersection of some normal affine ring with some function field in a generalized form (Theorems 2.1, 2.2).

In this article, by a *ring*, we mean a commutative ring with identity. By a *normal ring*, we mean an integral domainn which is integrally closed in its field of fractions. The *derived normal ring* of an integral domain A means the integral closure of A in the field of fractions of A. When we say that A is an *affine ring* over a ring B, we assume always that B is a noetherian integral domain, and A is a finitely gerated integral domain over B. By a *function field* over B, we mean the field of fractions of some affine ring over B.

1. Preliminaries

By a *discrete valuation*, we mean an additive valuation whose value group is isomorphic to $\boldsymbol{Z}$. Hence, a discrete valuation ring is a rank one discrete valuation ring.