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METRICAL THEORY FOR FAREY CONTINUED FRACTIONS

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1. Introduction

By making fundamental use of the Farey shift map and employing infinite (but σ-finite) measures together with the Chacon-Ornstein ergodic theorem it is possible to find new metrical results for continued fractions. Moreover this offers a unified approach to several existing theorems.

The application of ergodic theory to the study of continued fractions began with the Gauss transformation, G: $[0,1] \mapsto [0,1]$,

$$G(x)=\begin{cases} \dfrac{1}{x}-\left[\dfrac{1}{x}\right], & x\neq 0 \\ 0, & x=0, \end{cases}$$

which is ergodic with respect to the Gauss measure μ_g, where

$$\mu_g(B)=\frac{1}{\log 2}\int_B \frac{1}{1+x}dx$$

for any Borel subset B of $[0,1]$. H. Nakada [11] extended G to the 2-dimensional case. Let $\tilde{G}$: $[0,1]\times[0,1] \mapsto [0,1]\times[0,1]$ be defined to be

$$\tilde{G}(x,y)=\left(G(x),\frac{1}{a_1+y}\right)$$

where $a_1=\left[\dfrac{1}{x}\right]$. The absolutely continuous invariant measure of $\tilde{G}$, $\tilde{\mu}_g$, is given by

$$d\tilde{\mu}_g=\frac{1}{\log 2}\cdot\frac{dxdy}{(1+xy)^2}.$$

Then the dynamical system $([0,1]\times[0,1],\mathscr{B}_2,\tilde{\mu}_g,\tilde{G})$ is the natural extension of $([0,1),\mathscr{B}_1,G)$ where $\mathscr{B}_n$ is the Borel algebra of $\boldsymbol{R}^n$. Hence $\tilde{G}$ is ergodic with respect