Xiao, Y.
Osaka J. Math.
33 (1996), 895–913

HAUSDORFF MEASURE OF THE SAMPLE PATHS OF GAUSSIAN RANDOM FIELDS

YIMIN XIAO

(Received November 17, 1995)

1. Introduction

Let $Y(t)$ $(t\in \boldsymbol{R}^N)$ be a real-valued, centered Gaussian random field with $Y(0)=0$. We assume that $Y(t)$ $(t\in \boldsymbol{R}^N)$ has stationary increments and continuous covariance function $R(t,s)=EY(t)Y(s)$ given by

$$R(t,s)=\int_{\boldsymbol{R}^N}(e^{i\langle t,\lambda\rangle}-1)(e^{-i\langle s,\lambda\rangle}-1)\Delta(d\lambda), \tag{1.1}$$

where $\langle x,y\rangle$ is the ordinary scalar product in $\boldsymbol{R}^N$ and $\Delta(d\lambda)$ is a nonnegative symmetric measure on $\boldsymbol{R}^N\backslash\{0\}$ satisfying

$$\int_{\boldsymbol{R}^N}\frac{|\lambda|^2}{1+|\lambda|^2}\Delta(d\lambda)<\infty. \tag{1.2}$$

Then there exists a centered complex-valued Gaussian random measure $W(d\lambda)$ such that

$$Y(t)=\int_{\boldsymbol{R}^N}(e^{i\langle t,\lambda\rangle}-1)W(d\lambda) \tag{1.3}$$

and for any Borel sets A, $B\subseteq \boldsymbol{R}^N$

$$E(W(A)\overline{W(B)})=\Delta(A\cap B) \quad\text{and}\quad W(-A)=\overline{W(A)}.$$

It follows from (1.3) that

$$E[(Y(t+h)-Y(t))^2]=2\int_{\boldsymbol{R}^N}(1-\cos\langle h,\lambda\rangle)\Delta(d\lambda). \tag{1.4}$$

We assume that there exist constants $\delta_0>0$, $0<c_1\le c_2<\infty$ and a non-decreasing, continuous function $\sigma:[0,\delta_0)\to[0,\infty)$ which is regularly varying at the origin with index α $(0<\alpha<1)$ such that for any $t\in\boldsymbol{R}^N$ and $h\in\boldsymbol{R}^N$ with $|h|\le\delta_0$

$$E[(Y(t+h)-Y(t))^2]\le c_1\sigma^2(|h|). \tag{1.5}$$