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ON UNIQUENESS PROBLEM FOR LOCAL DIRICHLET FORMS

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1. Introduction

Let X be a locally compact separable metric space and let m be a positive Radon measure on X with everywhere dense support. Let $(\mathscr{E},\mathscr{F})$ be a regular Dirichlet space satisfying the strong local property, i.e., $\mathscr{E}(u,v)=0$ if u is constant on a neighbourhood of the support of the measure $|v|\cdot m$. Then, the form $\mathscr{E}$ can be written as

$$\mathscr{E}(u,u)=\frac{1}{2}\int_X d\mu_{\langle u\rangle}, \qquad u\in\mathscr{F},$$

where $\mu_{\langle u\rangle}$ is the *energy measure* of $u\in\mathscr{F}$ (cf. §3.2 in [7]). We say that a function u is *locally in* $\mathscr{F}$ ($u\in\mathscr{F}_{loc}$ in notation) if, for any relatively compact open subset G of X, there exists a function $w\in\mathscr{F}$ such that $u=w$ m-a.e. on G. Because of the strong locality of $(\mathscr{E},\mathscr{F})$, the energy measure $\mu_{\langle u\rangle}$ can be defined for $u\in\mathscr{F}_{loc}$.

A pseudo metric ρ on X associated with $(\mathscr{E},\mathscr{F})$ is defined by

$$(1) \qquad \rho(x,y)=\sup\{u(x)-u(y): u\in\mathscr{F}_{loc}\cap C(X),\ \mu_{\langle u\rangle}\leq m\},$$

where $\mu_{\langle u\rangle}\leq m$ means that the energy measure $\mu_{\langle u\rangle}$ is absolutely continuous with respect to m with Radon-Nikodym derivative $\dfrac{d\mu_{\langle u\rangle}}{dm}\leq 1$ m-a.e. The pseudo metric ρ is called *intrinsic metric* and its properties has been investigated by Biroli and Mosco [1] and Sturm [17], [18]. Now, we make the following:

ASSUMPTION A. ρ is a metric on X and the topology induced by it coincides with the original one. Moreover, (X,ρ) is a complete metric space.

The objective of this paper is to show the uniqueness of the extensions of $(\mathscr{E},\mathscr{F})$ under Assumption A. In §2, we shall prove that if $(\mathscr{E},\mathscr{F})$ fulfills Assumption A, then it has a unique extension in Silverstein's sense (Theorem 2.2), which was introduced in [14] in order to classify the symmetric Markov semigroups dominating