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GEVREY REGULARIZING EFFECT FOR NONLINEAR SCHRODINGER EQUATIONS

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1. Introduction

We consider the following Cauchy problem of nonlinear Schrödinger equations in n space dimensions,

$$\begin{cases} Lu \equiv i\partial_t u + \Delta u = f(t,x,u), \\ u(0,x) = \phi(x), \end{cases} \tag{1.1}$$

where $\Delta = \Sigma_{j=1}^{n} \partial^2 / \partial x_j^{\,2}$ and $f(t,x,u)$ is a complex valued function of Gevrey class in $(t,x,u) \in R \times R^n \times C$. We study the regularizing effect for (1.1). In what follows, we show that if the initial data ϕ is in some Gevrey class of order s with respect to $x \cdot \nabla_x$, then the solution u is in Gevrey class of order $\max(s/2,\ 1)$ with respect to x.

Concering the regularizing effect for dispersive equations, many works have been done ([1], [2], [5], [6], [7], [8], [9]). All the above works treat regularizing effects with respect to Sobolev spaces. In [4], N. Hayashi and one of the authors treat regularity in time for nonlinear Schrödinger equations. They have shown that if the initial data is in Gevrey class of order s (≥ 1) with respect to $x \cdot \nabla$ and ∇, then the solution is in Gevrey class of order s in space-time variables for $t \neq 0$. In [3], A. de Bouard, N. Hayashi and one of the authors treat Gevrey regularizing effect for nonlinear Schrödinger equatons in one space dimension and Korteweg-de Vries equation. They have shown that if the initial data is in Gevrey class of order s (≥ 1) with respect to $x \cdot \nabla$ and ∇, then the solution is in Gevrey class of order $\max(1, s/2)$ (or $\max(1, s/3)$ for KdV) with respect to the space variable for $t \neq 0$. We extend their results to the case that the nonlinear term is not polynomial, and for the local property, we extend their results to the case of higher space dimensions.

We introduce some notation and some function spaces to state the result precisely. Let $H^m(\Omega)$ denote Sobolev space of order m with respect to L^2 for an open set Ω in R^n. For simplicity we write $H^m = H^m(R^n)$. For a vector field Q with analytic coefficients and for a positive number M, we define a function space of Gevrey class $G^s_M(Q; H^m)$ in R^n as follows: