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KONTSEVICH INVARIANT FOR LINKS IN A DONUT AND LINKS OF SATELLITE FORM

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1. Introduction

In [9] Vassiliev, by investigating the complement of the discriminant of the space of knots, introduced the notion of finite type invariants (or Vassiliev invariants) of knots. Later, Birman and Lin [2] showed that the Vassiliev invariants can be characterized by the nilpotency with respect to the crossing changes. It turned out that the coefficients of many polynomial invariants, including quantum invariants, are of finite type. Later, Kontsevich [5], [1] modified the construction [4] of invariants for braids, which uses monodromy representations of formal connections on the configuration space, to get an invariant of knots. This invariant is called the universal Vassiliev-Kontsevich (VK) invariant, since one can get all the finite type invariants from it.

In this paper, we construct an isotopy invariant $\hat{Z}_f^{donut}$: {framed oriented links in a donut} $\rightarrow \mathscr{A}(F)$. For an arbitrary surface F, we denote by $\mathscr{A}(F)$ the space of chord diagrams on F in the sense of N.E. Reshetikhin [8]; $\mathscr{A}(F)$ is a $\boldsymbol{C}$-vector space formally generated by all the homotopy types of Feynman diagrams drawn on F with the relation called the four-term relation. Such an invariant is constructed independently in [3].

By using $\hat{Z}_f^{donut}$, we shall give a formula for satellite links of framed oriented links in $\boldsymbol{R}^3$. Let L_1 be a framed link in $\boldsymbol{R}^3$, K its component, $\hat{Z}_f(L_1)$ the VK invariant of L_1, and $\hat{Z}_f((L_1,K))$ the VK invariant of L_1 with specified circle corresponding to K. Let L_2 be a framed link in a donut, $(L_1,K)\diamond L_2$ the satellite obtained by substituting the donut containing L_2 into a tubular neighborhood of the component K of the framed link L_1 in $\boldsymbol{R}^3$ by a standard way. Let $\hat{Z}_f((L_1,K))\diamond\hat{Z}_f^{onut}(L_2)$ be the satellite obtained by modifying the distinguished component of $\hat{Z}_f((L_1,K))$ by $\hat{Z}_f^{donut}(L_2)$, which is introduced later. Then we have the following formula.

$$\hat{Z}_f((L_1,K)\diamond L_2)=\hat{Z}_f((L_1,K))\diamond\hat{Z}_f^{donut}(L_2).$$

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1) The author passesd away on August 14, 1995.