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HANDLEBODY DECOMPOSITIONS OF 4-MANIFOLDS AND TORUS FIBRATIONS

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1. Introduction

The purpose of this paper is to prove the following

Theorem A. *Let M be a closed smooth 4-manifold which has a handlebody decomposition of this type:*

$$M=H^0\cup aH^2\cup bH^3\cup H^4$$

where $b\leq 1$. Then M admits a torus fibration over the 2-sphere $f\colon M\to S$. The projection f is smooth except at a point $\in M$, and f has only one singular fiber.

In this theorem, a and b are the numbers of 2 and 3 handles, respectively. The type of the singular fiber in the above fibration is not necessarily 'good' in the sense of [7].

Theorem A was first announced in 1982 in [6]. (See also [7], [8].) The main reason for the long delay in publishing the proof is, of course, the author's laziness. But a reason was partly because the author was not fully convinced of the usefulness of the result; the variety of the singular fibers appearing in the construction seemed quite uncontrolable. As a matter of fact, such wide variety was a key to the proof of the existence theorem. Recently, the author received an enquiry from Daniel Ruberman about the proof. In trying to answer him, the author found a new example of a smooth torus fibration of S^4 over S^2 applying the general construction in this paper to S^4. Also, he found that, if $H_2(M;\boldsymbol{Z})\neq\{0\}$, we can arrange so that the general fiber is not homologous to zero in M (Theorem B in Section 3). He hopes that these improvements might justify this late publication of the proof. The author thanks D. Ruberman, whose enquiry gave him an opportunity to publish this paper.

2. Multiple fibered links

We begin by recalling 'multiple fibered links' from [6], [8]. Let L^3 be an oriented closed 3-manifold.