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THE K_*-LOCAL TYPE OF THE ORBIT MANIFOLD $(S^{2m+1}\times S^l)/D_q$ BY THE DIHEDRAL GROUP D_q

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Introduction

For a given CW-spectrum E there is an associated E-homology theory $E_*X=\pi_*(E\wedge X)$. A CW-spectrum Y is called E_*-local if any E_*-equivalence $A\to B$ induces an isomorphism $[B,Y]_*\cong[A,Y]_*$. For any CW-spectrum X there exists an E_*-equivalence $\iota_E: X\to X_E$ such that X_E is E_*-local. X_E is called the E_*-localization of X. Let KO and KU be the real and the complex K-spectrum respectively. There is no difference between the KO_*- and KU_*-localizations, and so we denote by S_K the K_*-localization of the sphere spectrem $S=\Sigma^0$. According to the smashing theorem [2, Corollary 4.7] the smash product $S_K\wedge X$ is actually the K_*-localization of X for any CW-spectrum X.

In this note we shall be interested in the K_*-local type of certain orbit manifolds $D(q)^{m,l}$ introduced as a filtration of a classifying space of the dihedral group D_q in [8]. The manifold $D(q)^{m,l}$ is defind as follows: Let $q\geq 3$ be an odd integer, and D_q the dihedral group generated by two elements a and b with relations $a^q=b^2=abab=1$. Consider the unit spheres S^{2m+1} and S^l in the complex $(m+1)$-space C^{m+1} and the real $(l+1)$-space R^{l+1}. Then D_q operates freely on the product space $S^{2m+1}\times S^l$ by

$$a\cdot(z,x)=(z\exp(2\pi\sqrt{-1}/q),x),\qquad b\cdot(z,x)=(\bar{z},-x)$$

where $\bar{z}$ is the conjugate of z. The associted topological quotient spaces

$$D(q)^{2m+1,l}=(S^{2m+1}\times S^l)/D_q=(L(q)^{2m+1}\times S^l)/Z_2,$$
$$D(q)^{2m,l}=(L(q)^{2m}\times S^l)/Z_2\subset D(q)^{2m+1,l}$$

are defined where $L(q)^{2m+1}=L^m(q)$ is the $(2m+1)$-dimensional lens space mod q and $L(q)^{2m}=L_0^m(q)$ its $2m$-skeleton.

The group $KU^0D(q)^{m,l}$ is decomposed to a direct sum of KU^0-groups of suspensions of stunted lens spaces mod q and mod 2 (cf. [5, Theorem 3.9]). Moreover KO^0- and J^0-groups of $D(q)^{m,l}$ have a quite similar direct sum decomposition (cf. [10] or [7]). In section 1 we shall show that $D(q)^{m,l}$ itself has