Shibuya, T.
Osaka J. Math.
33 (1996), 751–760

SELF Δ-EQUIVALENCE OF RIBBON LINKS

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(Received June 8, 1995)

1. Introduction

Throughout this paper, knots and links mean tame and oriented ones in an oriented 3-space R^3.

Several properties of local moves of links were studied in many papers, for example, homotopy of cobordant links in [3], [4] and boundary links in [1], [5], Δ-equivalence of links in [6], [7] and self #-equivalences of links in [8], [9] and [10].

In this paper, we investigate the self Δ-equivalence of ribbon links.

For a link ℓ, let B^3 be a 3-ball such that $\ell \cap B^3$ is a tangle illustrated in Fig. 1(a). The local move from Fig. 1(a) to (b) is called a Δ-*move.* Especially if these arcs are contained in the same component of ℓ, this move is called *a self* Δ *-move.* For two links ℓ and ℓ', if ℓ can be deformed into ℓ' by a finite se-

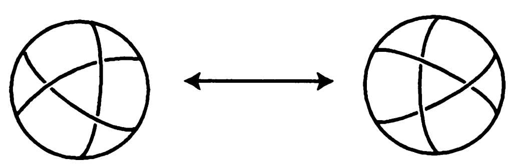

Fig. 1

quence of self Δ-moves, we say that ℓ and ℓ' are *self* Δ-*equivalent* or that ℓ is *self* Δ-*equivalent to* ℓ'.

An n-component link $\ell = k_1 \cup \cdots \cup k_n$ is called *a ribbon link* if there are n disks $\mathscr{C} = C_1 \cup \cdots \cup C_n$ in R^3 with $\partial\mathscr{C} = \ell$, $\partial C_i = k_i$, such that the singularity of $\mathscr{C}$, denoted by $\mathscr{S}(\mathscr{C})$, consists of mutually disjoint simple arcs of ribbon type, Fig. 2(b). For an arc β of $\mathscr{S}(\mathscr{C})$ in Fig. 2(b), the pre-images β^* and β'^* of β in Fig. 2(a) are called the *i-line* and the *b-line* respectively.

In this paper, we shall prove the following.

Theorem. *Any ribbon link is self* Δ-*equivalent to a trivial link.*

The author would like to thank the referee whose helpful comments greatly improved the presentations.