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THE THURSTON NORM AND THREE-DIMENSIONAL SEIBERG-WITTEN THEORY

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In October 1994 a flurry of activity started around a set of equations discovered by Seiberg and Witten [7]. By November there were three papers published on the Seiberg-Witten equations. One of those papers was Kronheimer and Mrowka's proof of the Thom conjecture [4]. The Thom conjecture was independently proved by Morgan, Szabo, and Taubes, and generalizations and variants of the Thom conjecture will be appearing soon [5], [2], [6]. The result in this paper is a three-dimensional version of the Thom conjecture.

The Thom conjecture is that an algebraic curve in $\boldsymbol{CP}^2$ realizes the minimal genus in a given homology class. In a general algebraic surface,

$$2g-2\geq|K\cdot F|+F\cdot F \tag{1}$$

when F is an embedded surface of genus g with positive self intersection and K is the canonical class. The same inequality holds when there is an algebraically non-trivial number of solutions to the equations

$$\begin{aligned} F_A^+ &= (\psi,\bar{\psi}) \\ \partial\!\!\!/_A\psi &= 0 \end{aligned} \tag{2}$$

on a line bundle with first Chern class K. The equations in (2) are the Seiberg-Witten equations, which we mentioned previously.

In 1986 Thurston defined a norm on the second homology of irreducible, atoroidal, 3 manifolds [9]. The norm of any non-trivial integral homology class is defined to be the minimum of $2g-2$ over all embedded surfaces representing the class. Our main theorem is exactly a lower bound on this quantity.

Theorem. *If the algebraic number of solutions to the 3D Seiberg-Witten equations is non zero, then*

$$2g-2\geq|c_1(L)\cap F|$$

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