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ON BORDISM INVARIANCE OF AN OBSTRUCTION TO TOPOLOGICAL EMBEDDINGS

To the memory of Maria Helena Derigi (Lene)

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1. Introduction

Let $f: M \to N$ be a continuous map of a smooth closed m-dimensional manifold into a smooth n-dimensional manifold with $k=n-m>0$. In [2] we have defined a primary obstruction $\theta_1(f) \in H_{m-k}(M; \boldsymbol{Z}_2)$ to the existence of a homotopy between f and a topological embedding. This homology class is represented by the closure of the self-intersection set of a generic smooth map (in the sense of Ronga [9]) homotopic to f and it has been shown that it is a homotopy invariant (for a precise definition of $\theta_1(f)$, see §2). Thus, if f is homotopic to a topological embedding (not necessarily locally flat), then $\theta_1(f)$ necessarily vanishes. (Nevertheless, we warn the reader that the vanishing of this primary obstruction does not necessarily imply the existence of a homotopy between f and a topological embedding.)

In this paper, we study the bordism invariance of the primary obstruction $\theta_1(f)$, which is a homotopy invariant. Here, two continuous maps f and $g: M \to N$ of a closed m-dimensional manifold M into a manifold N are said to be *bordant*, if there exist a compact (unoriented) $(m+1)$-dimensional manifold W with ∂W the disjoint union of two copies M_1 and M_2 of M and a continuous map $F: W \to N$ (called a *bordism* between f and g) with $F|M_1=f$ and $F|M_2=g$ (see [4] for the terminology). Note that, here, the domains of f and g are the same manifold.

Our main result of this paper is the following.

Theorem 1.1. *Let f and $g: M \to N$ be continuous maps of a smooth closed m-dimensional manifold into a smooth n-dimensional manifold with $k=n-m>0$. Suppose that $H^{m-k}(M; \boldsymbol{Z}_2)$ is generated by the elements of the form $w_{i_1}(M) \cup \cdots \cup w_{i_s}(M)$ with $i_1+\cdots+i_s=m-k$, where $w_j(M)$ denotes the j-th Stiefel-Whitney class of M. Then if f and g are bordant, then $\theta_1(f)=\theta_1(g)$.*

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