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TOTALLY GEODESIC HYPERSURFACES OF NATURALLY REDUCTIVE HOMOGENEOUS SPACES

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1. Introduction

Totally geodesic submanifolds of Riemannian symmetric spaces have been well investigated and it has been shown that they have beautiful and fruitful properties. In particular, due to the (M_+, M_-)-theory by B.Y. Chen and T. Nagano [1] this subject has made great progress. Naturally reductive homogeneous spaces are known as a natural generalization of Riemannian symmetric spaces. K. Tojo [6] investigated totally geodesic submanifolds of naturally reductive homogeneous spaces and obtained a necessary and sufficient condition of their existence. We will recall his result in section 3. Moreover he implicitly made the following conjecture.

Conjecture. *If a simply connected irreducible naturally reductive homogeneous space M admits a totally geodesic hypersurface, then M has constant sectional curvature.*

The conjecture is regarded as a generalization of the result which was shown in the case of Riemannian symmetric spaces by B.Y. Chen and T. Nagano [1]. K. Tojo gave an affirmative answer to the conjecture in the case that $\dim M = 3$, 4 and 5 [6] and in the case that M is a normal homogeneous space [7]. We shall prove that the conjecture above is true.

Main Theorem. *If a simply connected irreducible (as a Riemannian manifold) naturally reductive homogeneous space M admits a totally geodesic hypersurface, then M has constant sectional curvature.*

We shall discuss the irreducibility of naturally reductive homogeneous spaces in Section 2 and prove the main theorem in Section 3.

2. Irreducibility of naturally reductive homogeneous spaces

We first recall basic definitions and properties of naturally reductive