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FAMILIES OF SMOOTH *k*-GONAL CURVES WITH ANOTHER FIXED PENCIL

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0. Introduction

The actors of this paper are the same as the ones in [3], but the problems and methods are completely different (empty intersection). The actors are smooth curves, C, with 2 fixed pencils, say a g_k^1 and a g_b^1, which do not exist on curves with general moduli and that induce a birational morphism from C to a curve Y on a quadric surface $Q:=\boldsymbol{P}^1\times\boldsymbol{P}^1$, Y of bidegree (k,b). Indeed, while in [3] we studied a fixed such C, here we will study suitable families of such curves C.

In this paper we will work always in characteristic 0. In the first section we will prove (using very strongly [10]) the following result.

Theorem 0.1. *For all integers k, b, n with $0\leq n\leq bk-b-k+1$ and either $k\geq 4$ and $b\geq 10$ or $k\geq 5$ and $b\geq 8$, the smooth scheme $W((k,b),n)$ parametrizing the set of all nodal irreducible curves in Q of bidegree (k,b) and with geometric genus $g:=bk-b-k+1-n$ is irreducible.*

This theorem shows the power of the method introduced in [10] and refined very much in [11].

In the second section we will give a first step toward the Brill-Noether theory of special divisors on the general such curve C with as image $Y\subset Q$ a nodal curve, i.e. a curve $Y\in W((k,b),n)$. Remember that such a Brill-Noether theory is still in its infancy for curves not with general moduli. For interesting results for the case of general k-gonal curves, see [6] and [2]. In section 2 we will prove the following Brill-Noether type result.

Theorem 0.2. *Fix integers g, k, b, r, d with $r\geq 2$, $4\leq k\leq b$, $2k-2\leq g\leq bk-b-k+1$, $(r+1)d<r(2k+r-1)$. Let $S(g;k,b)$ be the constructible subset of the moduli space M_g of smooth curves of genus g parametrizing the curves, C, with a fixed pair of pencils, the first of degree k and the second of degree b, inducing a birational morphism from C onto a curve $Y\subset Q:=\boldsymbol{P}^1\times\boldsymbol{P}^1$. Then $S(g;k,b)$ is irreducible and a general $C\in S(g;k,b)$ has no g_d^r, only finitely many g_k^1 and no g_m^1 with $m<k$. Furthermore, C has Clifford index $k-2$.*