Watanabe, A.
Osaka J. Math.
33 (1996), 629–635

NORMAL SUBGROUPS AND MULTIPLICITIES OF INDECOMPOSABLE MODULES

ATUMI WATANABE

(Received September 4, 1995)

Introduction

Let G be a finite group and $(K,\boldsymbol{o},F)$ be a p-modular system, where p is a prime number. We assume that K contains the $|G|$-th roots of unity and F is algebraically closed and we put $R=\boldsymbol{o}$ or F. For an R-free finitely generated indecomposable RG-module M and a normal subgroup N of G, let V be an indecomposable component of M_N, where M_N is the restriction of M to N. In this paper we give some results on the multiplicity of V as a component of M_N and from them we obtain properties of heights of indecomposable modules and irreducible characters. This study is inspired by Murai [8, 9].

Throughout this paper N is a fixed normal subgroup of G and v is the p-adic valuation such that $v(p)=1$. All RG-modules are assumed to be R-free of finite rank. For an indecomposable RG-module M, let $\mathrm{vx}(M)$ denote a vertex of M. As is well known $v(\mathrm{rank}_R M)\geq v(|G:\mathrm{vx}(M|)$. We refer to Feit[1, Chap.3] and Nagao-Tsushima [10, Chap.4] for the vertex-source theory in modular representations of finite groups.

1. p-parts of multiplicities

In this section we study the p-parts of multiplicities of indecomposable RN-modules in an indecomposable decomposition of M_N. The following is a key result of this paper.

Theorem 1. *Let V be a G-invariant indecomposable RN-module. Let M be an indecomposable RG-module with vertex Q and n be the multiplicity of V in an indecomposable decomposition of M_N. Then we have $v(n)\geq v(|G:QN|)$.*

Proof. Let L be a subgroup of G such that L/N is a Sylow p-subgroup of G/N and let

$$M_L=M_1\oplus M_2\oplus\cdots\oplus M_s,$$

where each M_i is an indecomposable RL-module. By Mackey decomposition