Sordoni, V.
Osaka J. Math.
33 (1996), 589–605

ON THE WELL POSEDNESS OF THE CAUCHY PROBLEM FOR A CLASS OF HYPERBOLIC OPERATORS WITH MULTIPLE INVOLUTIVE CHARACTERISTICS

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(Received July 24, 1995)

1. Introduction

Let $X \subset \boldsymbol{R}^{n+1} = \boldsymbol{R}_{x_0} \times \boldsymbol{R}^n_{x'}$, $x' = (x_1, x_2, \cdots, x_n)$ be an open set such that $0 \in X$ and let us consider a differential operator of order m with C^∞ coefficients:

$$P(x, D_x) = P_m(x, D_x) + P_{m-1}(x, D_x) + \cdots \tag{1.1}$$

where we denote by $P_{m-j}(x, D_x)$ the homogeneous part of order $m-j$ of P.

Let us suppose that:

(H_1) the hyperplane $x_0 = 0$ is non-characteristics for P and the principal symbol $p_m(x, \xi)$ is hyperbolic with respect to ξ_0.

In this paper we shall study the well posedness of the Cauchy problem in C^∞ for the operator P in some cases where $p_m(x, \xi)$ is not strictly hyperbolic but the set of multiple characteristics has a very special form, as we will specify further. (For a definition of correctly posed Cauchy problem in $X_0 = \{x \in X; x_0 < 0\}$ we refer to [5]).

We shall suppose that $p_m(x, \xi)$ vanishes exactly of order $m_1 \leq m$ on a smooth manifold Σ and that p_m is strictly hyperbolic outside Σ.

On Σ we make the following assumptions:

(H_2) for any point $\rho \in \Sigma$, there exists a conic neighborhood Ω of ρ and $d+1$ ($d < n$) smooth functions q_j, $j = 0, \cdots, d$, defined on $W =: \Omega \cup (-\Omega)$ and homogeous of degree one such that $\Sigma \cap W$ is given by

$$\{\rho \in W; q_0(\rho) = .. = q_d(\rho) = 0\} \tag{1.2}$$

with $\{q_i, q_j\}(\rho) = 0$ for any $\rho \in \Sigma \cap W$.

(Here we have set $-\Omega =: \{(x, \xi) \in T^*X \backslash 0; (x, -\xi) \in \Omega\}$).

Moreover, denoting by ω and $\sigma = d\omega$ the canonical 1 and 2 forms in T^*X we suppose that $dq_j(\rho)$ and $\omega(\rho)$ are linearly independent one forms and that $H_{x_0}(\rho)$