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CLOSURES OF ORBITS OF *C* AND *C** ACTIONS

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1. Introduction and statement of results

Let G be a 1-dimensional complex linear algebraic group. In [7] Horrocks has shown that when G is the additive group $\boldsymbol{C}$ of complex numbers acting regularly on a normal (complex) projective variety, say X, or an algebraic variety X which is algebraically locally factorial, then the closure C of each G-orbit of is nonsingular. Moreover, Mabuchi [9] has shown that, if C touches a 1-codimensional component in X of the set X^G of fixed points, then C is nonsingular and it intersect X^G transversally for any complex manifold X.

On the other hand, when $G=\boldsymbol{C}^*$, the multiplicative group of complex numbers, Horrocks [7] showed that on a variety X as above the closure of each orbit is locally irreducible. If x is any point of X, this is equivalent to saying that either $x\in X^G$, or $0(x)\neq\infty(x)$, where $0(x)=\lim_{t\to 0}t(x)$ and $\infty(x)=\lim_{t\to\infty}t(x)$ for $t\in\boldsymbol{C}^*$. Furthermore, this fact was generalized as follows (cf. [3], [4]). Let X be a complex manifold on which $G=\boldsymbol{C}^*$ acts biholomorphically and meromorphically. Then a sequence of points $x_1,\cdots,x_s$, $s\geq 1$, of X is said to *generate a quasi-cycle* if $x_i\in X-X^G$ for each i, and $\infty(x_i)$ and $0(x_{i+1})$ are contained in one and the same connected component of X^G for $1\leq i\leq s$, where i is counted modulo s, so that $s+1=1$ by definition. Then the result is: If X is a compact Kähler manifold, then there exists no sequence of points on X which generates a quasi-cycle.

Here, we say that a biholomorphic action of G on X is *meromorphic* if the morphism of complex spaces $G\times X\to X$ defining the action extends to a meromorphic map $\boldsymbol{P}\times X\to X$ with respect to the natural inclusion $G\hookrightarrow\boldsymbol{P}$, where $\boldsymbol{P}$ denotes the complex projective line. The purpose of this note is then to generalize the above results in the following two theorems:

Theorem 1.1. *Let X be a normal compact Kähler space (cf. §4) on which a 1-dimensional complex linear algebraic group G acts biholomorphically and meromorphically. Then the following hold:*
1) *Suppose that $G=\boldsymbol{C}^*$. Then there exists no sequence of points $x_1,\cdots,x_s\in X-X^G$ which generates a quasi-cycle.*
2) *Suppose that $G=\boldsymbol{C}$. Then the closure C of any orbit is nonsingular, and C intersect X^G transversally at a (single) point x, where the transversality means that*