

THE EXCHANGE PROPERTY OF QUASI-CONTINUOUS MODULES WITH THE FINITE EXCHANGE PROPERTY

KIYOICHI OSHIRO and S. TARIQ RIZVI

(Received October 4, 1994)

Mohamed and Müller showed in [5] that continuous modules have the exchange property. And, recently, they also showed in [6] that for nonsingular quasi-continuous modules, the finite exchange property implies the exchange property. However, it is still open whether this is true or not for any quasi-continuous modules ([5, Problem 2]). The purpose of this paper is to answer this problem in the affirmative. This, then also provides another instance of modules for which the existence of the finite exchange property implies that of the exchange property in reference to the longstanding open question posed in Crawley-Jonsson [1].

Discrete (=semiperfect) modules and quasi-discrete (=quasi-semiperfect) modules are dual to continuous modules and quasi-continuous modules, respectively. We note that discrete modules have the exchange property and, for quasi-discrete modules, the finite exchange property implies the exchange property. These results follows by summarizing following results :

(1) (Oshiro [10]) Every quasi-discrete module M has an indecomposable decomposition $M = \sum_I \oplus M_i$ such that $M' = \sum_I \{M_i | M_i: \text{completely indecomposable}\}$ satisfies the finite exchange property. So, if M is discrete then it satisfies the finite exchange property since $M = M'$.

(2) (Harada-Ishii [2], Yamagata [12], [13]) If a module has an indecomposable decomposition and satisfies the finite exchange property, then it satisfies the exchange property in direct sums of completely indecomposable modules.

(3) (Zimmermann-Huisgen and Zimmermann [11]) A module M satisfies the exchange property if and only if for any $P = M \oplus X = \sum_I \oplus M_i$ with each $M_i \simeq M$, there exists $M'_i < \oplus M_i$ for each $i \in I$ such that $P = M \oplus \sum_I \oplus M'_i$.

The reader is referred to Mohamed and Müller'Book [5] for the background of these results.