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A CONDITION ON LENGTHS OF CONJUGACY CLASSES AND CHARACTER DEGREES

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1. Introduction

In E. Bannai [1], the following condition on finite groups is investigated.

Let G be a finite group, $\mathrm{Irr}(G)=\{\chi_i\}_{1\leq i\leq k}$ be the set of all irreducible characters of G, and $\mathrm{Cl}(G)=\{C_i\}_{1\leq i\leq k}$ be the set of all conjugacy classes of G.

Condition. By suitable renumbering i,

$$\chi_i(1)^2=|C_i|, \text{ for } i=1, 2, \cdots, k.$$

We call this condition B-condition ("B" is due to E. Bannai). A few groups satisfying B-condition are known: abelian groups, Suzuki 2-groups $A(n, \theta)$(See [3, VIII.6.7 Example and §7]), ϕ_6, ϕ_{11} in [4], and groups isoclinic to them (For isoclinism, see [2]). In any case, they are nilpotent and their derived lengths are at most 2.

In this paper, we shall construct a family of groups satisfying B-condition. Our groups are, in a sense, generalizations of Suzuki 2-groups. By our examples, we can say that

Theorem. *Derived lengths of groups satisfying B-condition are unbounded.*

2. Construction of groups

Let $F=\mathrm{GF}(2^n)$ be the finite field of order 2^n, and let θ be an automorphism of F. We put, for a positive integer l and $a_1, a_2, \cdots, a_l\in F$,