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INFINITE MARKOV PARTICLE SYSTEMS WITH SINGULAR IMMIGRATION ; MARTINGALE PROBLEMS AND LIMIT THEOREMS

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1. Introduction

In the present paper we are mainly concerned with infinite Markov particle systems $(X_t, \boldsymbol{P}_\mu)$ with singular immigration associated with absorbing Brownian motion $(w^0(t), P_x^0)$ in a half space $H=\boldsymbol{R}^{d-1}\times(0, \infty)$, starting from $\mu\in\mathcal{M}^I$. Here $\mathcal{M}^I=\mathcal{M}^I(H)$ is the spece of all σ-finite counting measures $\mu\in\sum_n\delta_{x_n}$ on H. It is constructed out of infinitely many independent absorbing Brownian particles starting from points of the support of μ and another independent particles which immigrate uniformly from boundary at random time and move according to the excursion law Q^0. The immigration part is obtained as the limit by putting the starting points of independent absorbing Brownian particles which immigrate in H at random times, close to the boundary with infinite mass. From this construction the generator $\mathcal{L}$ of this process should be expressed as the sum of no immigration part $\mathcal{L}^0$ and immigration part $\mathcal{L}^I$. That is, for some suitable functional $F(\mu)$ of integer-valued discrete measures μ,

$$\mathcal{L}^0F(\mu)=\frac{1}{2}\sum_{k=1}^{d}\langle\mu, \mathcal{D}_k^2F(\mu\,;\ \cdot)\rangle,$$

$$\mathcal{L}^IF(\mu)=\frac{1}{2}\langle\tilde{m}, \mathcal{D}_dF(\mu\,;\ \cdot)|_{x_d=0+}\rangle,$$

where $\tilde{m}=d\tilde{x}$ is the Lebesgue measure on $\boldsymbol{R}^{d-1}$, $\mathcal{D}_k$ is a kind of differential operator defined as

$$\mathcal{D}_kG(\mu\,;\ x)=\lim_{h\to 0}\frac{1}{h}[G(\mu+\delta_{x_k(h)}-\delta_x\,;\ x_k(h))-G(\mu\,;\ x)]$$

with $x_k(h)=(x_1, \cdots, x_k+h, x_{k+1}, \cdots, x_d)$, and $\mathcal{D}_k^2=\mathcal{D}_k\circ\mathcal{D}_k$ for $k=1, 2, \cdots, d$. Note that if $G(\mu\,;\ x)=F(\mu)$, then

$$\mathcal{D}_kF(\mu\,;\ x)=\lim_{h\to 0}\frac{1}{h}[F(\mu+\delta_{x_k(h)}-\delta_x)-F(\mu)].$$